

AUTOMORPHISMS OF PAIRWISE COMBINATORIAL DESIGNS

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This is an abstract of the PhD thesis *Automorphisms of Pairwise Combinatorial Designs* written by Pádraig Ó Catháin under the supervision of Dr Dane Flannery at the School of Mathematics, Statistics, and Applied Mathematics, National University of Ireland, Galway and submitted in August 2011.

The thesis investigates group actions on certain families of pairwise combinatorial designs, in particular Hadamard matrices and symmetric $2-(4t - 1, 2t - 1, t - 1)$ designs.

A Hadamard matrix H is called cocyclic if a certain quotient of the automorphism group contains a subgroup acting regularly on the rows and columns of H . We develop an algorithm for constructing all CHMs of order $4t$ based on a known relation between CHMs and relative difference sets. This method is then used to produce a classification of all CHMs of order less than 40. This is an extension and completion of work of de Launey and Ito.

If H is a CHM developed from a difference set then the automorphism group of H is doubly transitive. We show that the only CHMs with non-affine doubly transitive automorphism group are those that arise from the Paley Hadamard matrices. As a corollary of this result, we show that twin prime power difference sets and Hall sextic residue difference sets each give rise to a unique CHM.

We classify all difference sets which give rise to Hadamard matrices with non-affine doubly transitive automorphism group. In the process, we uncover a new triply infinite family of skew-Hadamard difference sets.

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