Ireland's Participation in the 51st International Mathematical Olympiad

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The International Mathematical Olympiad (IMO) is the oldest and largest among all international academic competitions for secondary school students. The first IMO was held in Romania in 1959, with 7 participating countries. At the end of the 1970s, about 20 countries were regularly participating in the IMO. Since then, the number of participating countries monotonically increased to over 100 in Bremen last year.

From 2$^{\text{nd}}$ until 14$^{\text{nd}}$ July 2010, the 51$^{\text{st}}$ International Mathematical Olympiad took place in Astana (Kazakhstan). With 523 participants (47 of whom were girls) from 97 countries, the size of this IMO, which was the first ever held in central Asia, was similar to its two predecessors.

The Irish delegation consisted of six students (see Table 1), the Team Leader, Bernd Kreussler (MIC Limerick) and the Deputy Leader, Gordon Lessells (UL).

Name	School	Year
Colin Egan	Clonkeen College, Blackrock, Co. Dublin	6$^{\text{th}}$
Dmitri Tuchapsky	Christian Brothers College, Cork	6$^{\text{th}}$
Vicki McAvinue	St. Angela's Secondary School, Waterford	5$^{\text{th}}$
Mel O'Leary	Lucan Community College, Co. Dublin	6$^{\text{th}}$
Owen Binchy	Coláiste Iognáid, Galway	6$^{\text{th}}$
Kieran Cooney	CBS Charleville, Co. Cork	5$^{\text{th}}$

TABLE 1. The Irish contestants at the 51$^{\text{st}}$ IMO

1. Team selection and preparation

The IMO is the most prestigious mathematical problem solving contest for second level students in the world. Participation in this event is already considered to be a great honour. In order to be able to gain any marks at the IMO exams, it is not sufficient, even for the brightest of students, to rely solely on the Leaving Certificate Mathematics Syllabus (or the equivalent in other countries). Some countries are able to identify the most talented students at a very early age and organise long-term training programmes for them, whereas others offer very intense training programmes for the brightest of their students which have been selected through a nationwide multiple round contest.

In Ireland, we are currently able to identify students with exceptional mathematical talent in three different ways: top performance in the Junior Certificate Examination in Mathematics, or excellent results in the PRISM competition or recommendation by a person who has identified this talent (maths teachers, school principals).

At five different locations all over Ireland (UCC, UCD, NUIG, UL and NUIM), mathematical enrichment programmes are offered to the students who came to our attention through one of these sources. These classes run each year from December/January until April and are offered by volunteer academic mathematicians from these universities or nearby third-level institutions.

This year, participation in the training programme was down considerably. In previous years, but, unfortunately, not this year, the Department of Education and Skills and the State Examinations Commission provided information which enabled school principals to identify those, if any, of their students who were among the best performers in the country in Junior Certificate Mathematics. This practice was very helpful in attracting healthy numbers of good students, and we hope that it will be possible to have the practice restored in coming years. We then contacted schools directly, but this is less efficient, because school principals often do not know whom they should nominate and might be too busy to send on this information to mathematics teachers.

In order to give the students the opportunity to gain additional competition experience, the centre in Limerick organised the first Limerick Mathematical Olympiad, which was held on 4^{th} March, 2010. Later, the winner of this Olympiad secured a place on the

Irish IMO team. The hope is to make this an annual event with increasing participation which may help to awake the interest of a greater number of students in the enrichment programme or in mathematics in general. A similar event takes place in Galway, to mark the end of the NUI Galway Mathematics Enrichment course, and to give students a chance to participate in a mathematical problem-solving contest before the IrMO. As in Limerick, the winner of this year's Galway Olympiad became a member of Ireland's IMO team.

The selection contest for the Irish IMO team is the Irish Mathematical Olympiad (IrMO), which was held for the 23$^{\text{rd}}$ time on Saturday, 24$^{\text{th}}$ April, 2010. The IrMO contest consists of two 3-hour papers on one day with five problems on each paper. The participants of the IrMO, who normally also attend the enrichment classes, sat the exam at the same time in one of the five centres. This year, a total of 43 students took part in the IrMO. The top performer is awarded the Fergus Gaines cup; this year this was Colin Egan. The best six students (listed in order in Table 1) were invited to represent Ireland at the IMO in Astana.

As in previous years, the final preparation of our contestants for the IMO took place in special training camps in Limerick. Because the IMO in Astana started more than one week earlier than usual, there was not much time available between the end of the Leaving Certificate Examinations and the departure date for our students. Therefore, instead of a week-long camp we organised two shorter camps for the students. The first of these was held at MIC Limerick on 8/9 June 2010. As the LC Examinations started on these days, four of the six team members could not participate. In fact, this camp could be seen as the start of the preparation for IMO 2011. The participants included the two team members who did not sit their LC Examinations this year and six of the best students (chosen on the basis of their performance at the IrMO) who will be eligible to participate in future IMOs. The second camp, at which the six members of the Irish IMO team participated, was held at the University of Limerick from 28 to 30 June 2010. The camps were organised as usual in a very efficient way by Gordon Lessells. This year, the sessions with the students were directed by Mark Burke, Mark Flanagan, Eugene Gath, Donal Hurley, Kevin Hutchinson, Bernd Kreussler, Tom Laffey, Jim Leahy and Gordon Lessells.

Thanks to an initiative of Stephen Buckley, communication among students as well as between them and their trainers was facilitated

by a bulletin board. This web-based tool with restricted access was used to discuss attempts of solutions of olympiad-style problems between the weekly enrichment sessions and also between the IMO preparation camps.

2. Jury meetings – the problem selection

The Jury of the IMO, which is composed of the Team Leaders of the participating countries and a Chairperson who is appointed by the organisers, is the prime decision making body for all IMO matters. In particular it is responsible for choosing the six contest problems out of a shortlist of 30 problems provided by a problem selection committee, also appointed by the host country. This year's Chairman of the Jury was Professor Yerzhan Baissalov.

In the recent past, the location at which the Jury of the IMO resides was kept as secret as possible as one of the measures which should help to ensure that the contest problems will not become publicly known before the exams take place. This year, it was known in advance that the Jury members were supposed to arrive in Almaty (known as Alma-Ata until 1993), at a distance of about 950 km from Astana. Almaty was the capital of Kazakhstan until 1998 and is still an important cultural, scientific and business centre of Kazakhstan.

The Jury meetings which took place on 3–5 July were held in the Wellness Complex "Alatau" outside of Almaty, at the foothills of the Trans-Ili Alatau Mountains, the northernmost part of the Tien Shan mountain system. This beautiful and quiet location was the ideal place to think about the proposed contest problems. As my flight had arrived in Almaty shortly after midnight on the 2 July, I had plenty of time to get familiar with the shortlisted problems without being spoilt by the official solutions which were handed out on the next day after lunch.

After having obtained the official solutions, time was less plentiful, because during the Jury meetings on Sunday, 4^{th} July, the six contest problems had to be selected. Based on the longstanding experience of many Jury members and the calm direction of the Chairman, this task was completed before 4 p.m. on Sunday.

During the evening meeting, the final formulation of the six problems was discussed so that representatives of the language groups of the five official languages (English, French, German, Russian and

Spanish) were able to produce their versions over night. After approving these five versions at a Jury meeting on Monday morning, the rest of the day was available for the translation of the contest problems into 50 languages, in addition to the five mentioned before.

Because the Jury was to be flown to Astana, all exam papers had to be printed before departure. This task was finished in time for the very early departure to the airport at 3 a.m. on Monday, 6 July. The reason why the organisers were keen to finish the printing before departure is that even with the use of the PDF format, it may happen after changing to another PC, that problems are arising in viewing or printing a document which contains locally unusual fonts. As usual in recent years, the support provided by Matjaž Željko from Slovenia was invaluable for all IT related tasks at this IMO.

3. The problems

First Day. Problem 1. Determine all functions $f : \mathbb{R} \to \mathbb{R}$ such that the equality

$$f(\lfloor x \rfloor y) = f(x)\lfloor f(y) \rfloor$$

holds for all $x, y \in \mathbb{R}$. (Here $\lfloor z \rfloor$ denotes the greatest integer less than or equal to z.) *(France)*

Problem 2. Let I be the incentre of triangle ABC and let Γ be its circumcircle. Let the line AI intersect Γ again at D. Let E be a point on the arc BDC and F a point on the side BC such that

$$\angle BAF = \angle CAE < \tfrac{1}{2}\angle BAC.$$

Finally, let G be the midpoint of the segment IF. Prove that the lines DG and EI intersect on Γ. *(Hong Kong)*

Problem 3. Let $\mathbb{N}$ be the set of positive integers. Determine all functions $g : \mathbb{N} \to \mathbb{N}$ such that

$$\big(g(m) + n\big)\big(m + g(n)\big)$$

is a perfect square for all $m, n \in \mathbb{N}$. *(United States of America)*

Second Day. Problem 4. Let P be a point inside the triangle ABC. The lines AP, BP and CP intersect the circumcircle Γ of triangle ABC again at the points K, L and M respectively. The tangent to Γ at C intersects the line AB at S. Suppose that $SC = SP$. Prove that $MK = ML$. *(Poland)*

Problem 5. In each of six boxes $B_1, B_2, B_3, B_4, B_5, B_6$ there is initially one coin. There are two types of operation allowed:

Type 1: Choose a nonempty box B_j with $1 \leq j \leq 5$. Remove one coin from B_j and add two coins to B_{j+1}.

Type 2: Choose a nonempty box B_k with $1 \leq k \leq 4$. Remove one coin from B_k and exchange the contents of (possibly empty) boxes B_{k+1} and B_{k+2}.

Determine whether there is a finite sequence of such operations that result in boxes B_1, B_2, B_3, B_4, B_5 being empty and box B_6 containing exactly $2010^{2010^{2010}}$ coins. (Note that $a^{b^c} = a^{(b^c)}$.) *(Netherlands)*

Problem 6. Let $a_1, a_2, a_3, \ldots$ be a sequence of positive real numbers. Suppose that for some positive integer s, we have

$$a_n = \max\{a_k + a_{n-k} \mid 1 \leq k \leq n-1\}$$

for all $n > s$. Prove that there exist positive integers ℓ and N, with $\ell \leq s$ and such that $a_n = a_\ell + a_{n-\ell}$ for all $n \geq N$. *(Iran)*

4. The contest

The six Irish contestants, accompanied by the Deputy Leader, Gordon Lessells, arrived in Astana in the very early morning of Sunday, 4^{th} July. The early arrival one day ahead of schedule did help to minimise the costs for the flights and was very helpful for adjusting to the local time which is 5 hours ahead of Irish Summer Time.

Since 1998 Astana is the capital of Kazakhstan. An incredible feast of architecture has tripled the area covered by this city compared with its size before 1997. The contrast between the original part of Astana (known as Akmola between 1992, when Kazakhstan became independent, and 1998) and the newly created City could not be bigger. Many buildings in the old town look very run down and do not seem to be maintained properly anymore. At the same time, the contemporary architecture of the new City is very shiny, with a

great variety of forms, wide roads, expensive ultramodern shopping centres and the palace of the president, the pyramid shaped Palace of Peace and Reconciliation, the Bajterek tower (the new symbol of Astana) and a few more structures in exact alignment.

The Opening Ceremony took place on Tuesday morning at the Palace of Independence in Astana. In addition to the traditional parade of all participating teams and the usual speeches, we saw a nice and colourful presentation of traditional and modern music and dance from Kazakhstan.

On Tuesday afternoon the students were driven to their new accommodation in the Youth Centre "Baldauren", situated on the shore of Lake Shchuchye, 240 km north of Astana in a very scenic part of Kazakhstan. The six-hour bus trip with dinner at 10.30 p.m. was not the ideal preparation for the first exam next day. Baldauren was the venue for the exams and the centre for a variety of recreational and sight-seeing activities after the exams were over. The facilities available were of a high standard and much appreciated by the students. The Irish Team guide, Zuzaira, was particularly helpful throughout this time. The students returned to Astana for three nights at the end of the Olympiad.

On the day the students left for Astana, the members of the Jury checked in to Hotel "Duman" which the Students and Deputies had vacated in the morning. For the afternoon and evening, meetings were scheduled for the discussion of the detailed marking schemes for all six contest problems. Due to delay with the check-in procedure, the meetings were postponed to 8 p.m. in the evening. The meeting was adjourned at 10:15 p.m., because everybody was too tired from the very early start into this day. The finalisation of the marking schemes of three problems was postponed to the next morning, directly after the questions-and-answers session. The problem captains indicated that they intend to follow the marking schemes very strictly.

The two exams took place on the 7^{th} and 8^{th} of July, starting at 9 o'clock each morning. There were three venues for the exams, all located in the "Baldauren" resort. On each day, $4\frac{1}{2}$ hours were available to solve three problems. During the first 30 minutes, the students were allowed to ask questions if they had difficulties in understanding the formulation of a contest problem. A scan of these questions was sent to the hotel "Duman", where the printout was given to the relevant Team Leader. Each question was discussed

and answered in front of the Jury, so that equal standards were applied to all contestants. The approved answer was then sent back electronically to the exam location. The questions did not indicate that there was any major source of confusion in the formulation of the problems. With 26 questions on the first day and about 55 on day two, these were unusually short Q&A sessions.

On the evening of the 8^{th} of July an extraordinary Jury meeting took place at which the team of the Democratic Peoples Republic of Korea was disqualified for this year.

5. Marking and Coordination

The first scripts had arrived late Wednesday evening. A first reading on that night revealed that only Colin's solution of Problem 1 could be a candidate for full marks. Unfortunately, it turned out later that he didn't finish one case completely and so narrowly missed an Honourable Mention.

After the end of the second exam on Thursday, the Deputy Leaders were transferred to the hotel "Duman". Because the venue for the exams and the students' accommodation were far away from our hotel, there was no way to meet the students. The first time the leaders were able to briefly see their students was on Monday, 12^{th} July, during an impressive presentation of traditional Kazakh equestrian sports at the hippodrome "Kazanat", not far from Astana.

Gordon arrived at about 8 p.m. on Thursday. All our coordination sessions were scheduled for Friday and Saturday – the first one on Friday morning at 9 o'clock. This was a very tight programme and we were painfully missing the helping hand of an Official Observer during two almost sleepless nights.

The marking of the scripts at the IMO is undertaken by two independent groups. One group consists of the Team Leader, the Deputy Leader and, if available, the Official Observer. The second group consists of the coordinators, who were appointed by the local organisers. The two groups met according to a tight schedule which was distributed on the morning of the second exam day.

The coordination is a very important part of the IMO, because it is the well established method to fairly mark the scripts of the students from so many different nations. It is important for the representatives of the teams to be well prepared for each of the half-hour meetings with the coordinators. For example, a detailed explanation

of each step of a student's solution could be necessary or there might be the need to show that filling the gaps in a solution, which was not covered by the agreed marking scheme, does not require deep ideas and is easily carried out.

We found that the very young coordinators were very knowledgeable and always well prepared. In general they were very strictly adhering to the marking scheme and did not award points too generously. Other Team Leaders have experienced the coordination process in the same way so that we were sure that the points were distributed with equal measure in a fair way.

Name	P1	P2	P3	P4	P5	P6	total	ranking
Colin Egan	6	0	0	0	0	0	6	446
Owen Binchy	0	0	0	0	6	0	6	446
Kieran Cooney	0	0	0	0	2	0	2	485
Mel O'Leary	2	0	0	0	0	0	2	485
Vicki McAvinue	0	1	0	0	0	0	1	494
Dmitri Tuchapsky	0	0	0	0	1	0	1	494

Table 2. The results of the Irish contestants

Table 2 shows the results of the Irish contestants. A comparison with the results achieved by all other contestants reveals a weakness of our students in geometry (Problems 2 and 4). However, the Irish contestants have shown their true potential on Problem 5, where they scored well above average. To understand these anomalies, one has to take into account that Problem 5 does not fit into any standard pattern and so creativity was more important than experience. On the other hand, to be good in solving geometry problems requires a lot of experience which normally is acquired through continuous training during a couple of years.

The Jury tries to choose the problems in such a way that Problems 1 and 4 are easier than Problems 2 and 5. Problems 3 and 6 are usually designed to be the hardest problems. Table 3 shows that this year's results fit very well into this pattern, whereby Problem 5 was a bit harder than anticipated.

If the medal cut-offs are taken as an indicator, it seems that achieving high marks at this IMO has been harder than in previous years. But this might also be related to a less generous marking process. Gold medals were awarded to 47 students who scored at

	P1	P2	P3	P4	P5	P6
0	39	223	429	84	353	471
1	17	93	48	2	85	14
2	27	23	10	10	26	4
3	16	8	4	47	3	3
4	34	2	4	2	0	0
5	35	4	4	4	2	6
6	54	2	2	2	11	4
7	295	162	16	366	37	15
average	5.453	2.586	0.464	5.348	0.930	0.368

TABLE 3. For each problem, how many contestants have got how many points

least 27 points, 104 students with points in the range 21–26 got silver medals and each of the remaining 115 students who scored at least 15 points was awarded a bronze medal.

Although the IMO is a competition for individuals only, it is interesting to compare the total scores of the participating countries. This year's top teams were from China (197 points), Russia (169 points) and the USA (168 points). Ireland, with 18 points in total, finished in 90^{th} place. Only one student, Zipei Nie from China, achieved the perfect score of 42 points. The detailed results and statistics can be found on the official IMO website `http://www.imo-official.org`.

6. European Girls' Event

The Leader of the UK team, Geoff Smith, organised a meeting on Sunday, 11^{th} July, where he announced the launch in 2012 of a European Girls' Event. The original plan, he said (after apologising not to be a woman), was to have a Girls' Mathematical Olympiad. There was an email discussion among some European Team Leaders about this idea before they travelled to the IMO. This original idea was received with some scepticism. A basis of this scepticism might be that in most European countries separate girls' education is not common.

The discussion showed that an initiative which could help to increase female participation in the IMO and to encourage more girls to engage in mathematical problem solving is very welcome. On the other hand, to create a girls only competition similar to the IMO could be seen to be discriminatory. It seemed that the consensus

was that the main focus of such an event should be on educational activities which boost the participants' abilities in competitions like the IMO and which help to develop girls' interest in mathematics. Part of such a training camp-like event could well be a competitive exam.

What the exact flavour of it will be depends on the organisers of the first event of this kind. According to Geoff, Murray Edwards College Cambridge and the UK Mathematical Trust will host it in Easter 2012. The hope is that this will be the start of an annual event, held in different European countries in the future.

7. Outlook

The closing ceremony took place on Tuesday, 13$^{\text{th}}$ July, at the Palace of Independence in Astana. Speeches were given, the medals were awarded and folklore and contemporary music performances in between made it a nice event. At the end, the IMO banner was passed on to the delegation from the Netherlands, where the 52$^{\text{nd}}$ IMO will be held from 16$^{\text{th}}$ until 24$^{\text{th}}$ July 2011.

The next countries to host the IMO will be

2011	Netherlands	`http://www.imo2011.nl/`
2012	Argentina	

Standard procedure would have been to formally decide this year about the location of the IMO 2013. However, no such decision could be made, because no country has expressed its interest in organising the IMO that year.

At one of the Jury meetings, Nazar Agakhanov, Leader of the team from Russia, was elected as new chairman of the IMO Advisory Board. The outgoing chair, József Pelikán from Hungary, who has served for the past eight years, was thanked for his work with standing ovation.

During the final Jury meetings a discussion was started about reforming some IMO procedures. This includes the problem selection process and the regulations to deal with allegations of fraud. Within the next 11 months these issues should be discussed by all those interested in IMO matters.

8. Conclusions

Due to the recession, the funding provided by the Department of Education and Skills was not sufficient to send a full team of six students. For the first time in 14 years, no funding was available to send an Official Observer who accompanies and supports the Team Leader. The work done by the Leader and Deputy Leader, in particular when marking the scripts of the students is enormous. There is a real difference if an observer is available and if so, this helps to ensure that the best possible results for the Irish students could be achieved. Without an observer there is just not enough time to go through all solutions of all our contestants with the same care. Even in the problem selection stage, the helping hand of an observer is very useful to get a better picture of the difficulties involved in the shortlisted problems. Therefore, efforts have to be increased to get better funding in the future.

The results of the Irish Team show that our students have much less experience in problem solving than the majority of the contestants from other nations. This becomes obvious by looking at Problem 4, for which 366 students (more than 70 percent) got the full score of 7 points. Among the top 50 countries, only Denmark with 15 points has scored less than 26 for Problem 4 as a team. There are only two other teams who did not score any points for Problem 4.

Improved Irish performance at the IMO requires a solid foundation in basic geometry. We have very good geometry teachers in all the five training centres in Ireland, but our students are exposed to problems from elementary geometry too late. Building up experience in solving mathematical problems, not only from geometry, needs time. The way forward seems to be to get students in their Junior Cycle interested in such activity. The PRISM competition (`http://www.maths.nuigalway.ie/PRISM/`) is a promising and valuable activity which may help to attract younger students.

Because almost all participating countries have performed better than our team on the two geometry questions, we need to find answers to the following questions. Can we improve the training in geometry in our enrichment programmes? Is the weakness of our students in geometry caused by the Irish school curriculum or how the curriculum is put into practice? What else could be the underlying reasons?

It would be alarming if the cause would be found in relation to the curriculum or its implementation, because elementary geometry is a traditional subject trough which primary and secondary school children could be and indeed were successfully introduced to clean logical thinking, mathematical proof and problem solving. As there are almost no prerequisites needed in order to do geometry, this is the ideal subject through which such skills can be developed from an early age.

On the positive side, it is a very encouraging fact that three of our students scored on Problem 5. Only 14 of the top 50 countries have got more than the 9 points our team has scored for this problem. I take this as an indication that it is the experience and not the talent or creativity our students are lacking.

At a time where interest in mathematics and science courses at third level institutions is on the increase, it should be possible to get secondary school teachers interested in fostering their mathematically talented students. To enhance skills related to logical thinking, mathematical proof and problem solving, certainly would be a very valuable contribution to the education of future generations of students who wish to study mathematics at university.

I suggest that we should try to initiate a platform or network for those teachers who are willing to run regular training sessions with focus on mathematical problem solving in their schools for students in their Junior Cycle. The trainers from the five training centres could then support these teachers with their experience.

Those students who really engage in mathematical problem solving activities will experience the satisfaction of success whenever they solve a problem having spent some hours tackling it. Such satisfaction could be an important motivation in continuing to think independently about mathematical problems. And this is what a successful IMO participant as well as a good student of mathematics needs.

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The essential part of the preparation of the contestants is the work with the students done in the enrichment programmes at the five universities. This work is carried out for free by volunteers in their spare time. Thanks go to this year's trainers at the five Irish centres:

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At UCD: David Brink, Mark Flanagan, Marius Ghergu, Kevin Hutchinson, Tom Laffey, Gary McGuire and Brundaban Sahu.

At NUIG: Javier Aramayona, Graham Ellis, Neil Humphries, Niall Madden, David O'Keeffe, Edwin O'Shea, Rachel Quinlan, Ray Ryan and Jerome Sheahan.

At UL: Mark Burke, Eugene Gath, Bernd Kreussler, Jim Leahy and Gordon Lessells.

At NUIM: Gavin Armstrong, Sonia Balagopalan, Stefan Bechtluft–Sachs, Caroline Brophy, Stephen Buckley, Peter Clifford, Katarina Domijan, Kurt Falk, David Malone, John Murray, Anthony G. O'Farrell, Lars Pforte, Adam Ralph and David Redmond.

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