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Methods of Ascent and Descent in Multivariable Spectral Theory

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This is an abstract of the PhD thesis *Methods of ascent and descent in multivariable spectral theory* written by Derek Kitson under the supervision of Professor Richard M. Timoney at the School of Mathematics, Trinity College Dublin and submitted in June 2008.

In this thesis the classical notions of ascent and descent for an operator acting on a vector space are extended to arbitrary collections of operators. The resulting theory is applied to the study of joint spectra for commuting tuples of bounded operators acting on a complex Banach space. Browder joint spectra are constructed and shown to satisfy a spectral mapping theorem.

For a set A of operators on a vector space X we define the *ascent* $\alpha(A)$ and *descent* $\delta(A)$ as the smallest non-negative integers such that

$$N(A) \cap R(A^{\alpha(A)}) = \{0\} \text{ and } N(A^{\delta(A)}) + R(A) = X$$

where $N(A)$ denotes the joint null space, $R(A)$ the joint range space and A^k the set of all products of k elements. We show that the collection A has finite ascent and finite descent if and only if there exist A-invariant subspaces X_1, X_2 with $X = X_1 \oplus X_2$ such that the restriction of A to X_1 satisfies a nilpotent condition while A restricted to X_2 satisfies a bijectivity condition. Moreover, the ascent and descent of A are necessarily equal and determine X_1 and X_2 uniquely:

$$X_1 = N(A^r) \text{ and } X_2 = R(A^r) \text{ where } r = \alpha(A) = \delta(A).$$

For commuting n-tuples $\mathbf{a} = (a_1, \ldots, a_n)$ of bounded operators on a complex Banach space we define a Browder joint spectrum

$$\sigma_b(\mathbf{a}) = \{\lambda \in \mathbb{C}^n : \mathbf{a} - \lambda \notin \mathcal{B}\}$$

where $\mathcal{B}$ is the collection of commuting Fredholm n-tuples with finite ascent and finite descent. This Browder joint spectrum is smaller than the Taylor-Browder spectrum of [1] but contains the upper and lower semi-Browder spectra of [3]. We show that this Browder joint spectrum is compact-valued, has the projection property and consequently satisfies a spectral mapping theorem:

$$\sigma_b(f(\mathbf{a})) = f(\sigma_b(\mathbf{a}))$$

for all mappings f holomorphic on the Taylor spectrum of $\mathbf{a}$. We also give a characterisation

$$\sigma_b(\mathbf{a}) = \bigcap_{\mathbf{r} \in \mathcal{R}} \sigma_\pi(\mathbf{a} + \mathbf{r}) \cup \sigma_\delta(\mathbf{a} + \mathbf{r})$$

where σ_π and σ_δ denote respectively the joint approximate point and defect spectra and $\mathcal{R}$ denotes the collection of all commuting n-tuples of Riesz operators which commute with $a_1, \ldots, a_n$.

Analogous results are obtained for the Harte spectrum, the Taylor spectrum, the Slodkowski spectra $\sigma_{\pi,k} \cup \sigma_{\delta,l}$ and their split versions. We show that necessary and sufficient for a commuting n-tuple $\mathbf{a} = (a_1, \ldots, a_n)$ to be Taylor-Browder is that $\mathbf{a} = \mathbf{c} + \mathbf{s}$ where $\mathbf{c} = (c_1, \ldots, c_n)$ is a commuting tuple of compact operators, $\mathbf{s} = (s_1, \ldots, s_n)$ is Taylor-invertible and $c_i s_j = s_j c_i$ for all i, j.

Multivariable analogues of the notion of a *pole* and a *Riesz point* for an operator are introduced for commuting tuples $\mathbf{a} = (a_1, \ldots, a_n)$. We use poles to investigate a several variable version of N. Dunford's minimal equation theorem and Riesz points are used to characterise commuting tuples of Riesz operators. Applications to a multivariable Weyl's Theorem are considered.

References

[1] R.E. Curto and A.T. Dash, Browder Spectral Systems. *Proc. Amer. Math. Soc.*, 103(2):407-413, 1988.

[2] D. Kitson, Ascent and descent for sets of operators. *Studia Math.*, to appear.

[3] V. Kordula, V. Müller and V. Rakočević, On the Semi-Browder Spectrum. *Studia Math.*, 123(1):1-13, 1997.

Polynomials on Riesz Spaces

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This is an abstract of the PhD thesis *Polynomials on Riesz Spaces* written by John Loane under the supervision of Dr. Ray Ryan at the Department of Mathematics, National University of Ireland, Galway and submitted in December 2007.

Mathematicians have been exploring the concept of polynomial and holomorphic mappings in infinite dimensions since the late 1800's. From the beginning the importance of representing these functions locally by monomial expansions was noted. Recently Matos studied the classes of homogeneous polynomials on a Banach space with unconditional basis that have pointwise unconditionally convergent monomial expansions relative to this basis. More recently still Grecu and Ryan noted that these polynomials coincide with the polynomials that are regular with respect to the Banach lattice structure of the domain.

In this thesis we investigate polynomial mappings on Riesz spaces. This is a natural first step towards building up an understanding of polynomials on Banach lattices and thus eventually gaining an insight into holomorphic functions.

We begin in Chapter 1 with some definitions. A polynomial is defined to be positive if the corresponding symmetric multilinear mappings are positive. We discuss monotonicity for positive homogeneous polynomials and then give a characterization of positivity of homogeneous polynomials in terms of forward differences.

In Chapter 2 we show that, as in the linear case positive multilinear and positive homogeneous polynomial mappings are completely determined by their action on the positive cone of the domain and furthermore additive mappings on the positive cone extend to the whole space. We conclude by proving formulas for the positive part, the negative part and the absolute value of a polynomial mapping.

In Chapter 3 we prove extension theorems for positive and regular polynomial mappings. We consider the Aron-Berner extension for homogeneous polynomials on Riesz spaces.

In Chapter 4 we first review the Fremlin tensor product for Riesz spaces and then consider a symmetric Fremlin tensor product. We discuss symmetric k-morphisms and define the concept of polymorphism. We give several characterizations of k-morphisms in terms of these polymorphisms. Finally we consider orthosymmetric multilinear mappings.

References

[1] C. D. Aliprantis and O. Burkinshaw, *Positive Operators,* Pure and Applied Mathematics, 119. Academic Press, Inc., Orlando, FL, 1985. xvi+367 pp. ISBN: 0-12-050260-7.

[2] G. Buskes and A. van Rooij, *Bounded variation and tensor products of Banach lattices,* Positivity and its applications (Nijmegen, 2001). Positivity 7 (2003), no. 1-2, 47–59.

[3] D. H. Fremlin, *Tensor products of Archimedean vector lattices,* Amer. J. Math. 94, 1972, 777–798.

[4] B. C. Grecu and R. A. Ryan, *Polynomials on Banach spaces with unconditional bases,* Proc. Amer. Math. Soc. 133 (2005), no. 4, 1083–1091.

A Theoretical Study of Spin Filtering and its Application to Polarizing Antiprotons

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This is an abstract of the PhD thesis *"A theoretical study of spin filtering and its application to polarizing antiprotons"* written by Domhnaill O'Brien under the supervision of Dr. Nigel Buttimore at the School of Mathematics, Trinity College Dublin and submitted in June 2008.

There has been much recent research into possible methods of polarizing an antiproton beam, the most promising being spin filtering, the theoretical understanding of which is currently incomplete. The method of polarization buildup by spin filtering requires many of the beam particles to remain within the beam after repeated interaction with an internal target in a storage ring. Hence small scattering angles, where we show that electromagnetic effects dominate hadronic effects, are important. All spin-averaged and spin-dependent electromagnetic cross-sections and spin observables for elastic spin 1/2 - spin 1/2 scattering, for both point-like particles and non-point-like particles with internal structure defined by electromagnetic form factors, are derived to first order in QED. Particular attention is paid to spin transfer and depolarization cross-sections in antiproton-proton, antiproton-electron and positron-electron scattering, in the low $|\, t \,|$ region of momentum transfer. A thorough mathematical treatment of spin filtering is then presented, identifying the key physical processes involved and highlighting the dynamical properties of the physical system. We present and solve sets of differential equations which describe the buildup of polarization by spin filtering in many different scenarios of interest. The advantages of using a lepton target are outlined, and finally a proposal to polarize antiprotons by spin filtering off an opposing polarized electron beam is investigated.