BOOK REVIEWS

Essential Topology

BY MARTIN D. CROSSLEY

Springer Undergraduate Mathematics Series. Springer-Verlag, London, 2005, x+224 pages.

reviewed by Ariel Blanco, Department of Pure Mathematics, School of Mathematics and Physics, Queen's University Belfast, Belfast BT7 1NN, a.blanco@qub.ac.uk

This textbook brings to the reach of second-year (possibly first-year) undergraduates some of the most fundamental ideas and results of the subject. Written in a very clear and lively style, the book takes the student from a naive approach to continuity to advanced topics like homotopy and homology theory. The numerous examples throughout the text, none complicated and all of them explained in a very clear and concise manner, should make the material easier to grasp by the student. As a rule, lengthy proofs are omitted throughout the book. Instead references are given to books that contain them.

The book splits naturally in two parts. The first part (Chapters 1–5) corresponds to a course on set-theoretic topology. After a brief introduction in Chapter 1, the author arrives to the definition of continuity in terms of open sets in Chapter 2. The notion of topological space is introduced in Chapter 3. Chapter 4 is devoted to the study of connectivity, compactness and the Hausdorff property. The concept of homeomorphism together with some important topological constructions such as disjoint unions, product and quotient spaces are treated in Chapter 5. This part of the book only requires from the reader some familiarity with the notion of continuity of a function as presented in Calculus, with equivalence relations and with matrix algebra. (Some familiarity with the geometry of $\mathbb{R}^3$ would be useful.)

The second part of the book (Chapters 6–11) corresponds to a course on algebraic topology. This part starts with the notion of homotopy equivalence which is developed in Chapter 6. Also Brouwer's Fixed-Point theorem and the Hairy Ball theorem are proved in this chapter. Chapter 7 is a brief account of the Euler number. Homotopy groups are studied in Chapter 8. Simplicial and singular homologies are treated in Chapters 9 and 10 respectively. In particular, the Mayer–Vietoris sequence is derived in Chapter 10, where the relation between homotopy and homology groups is discussed. Lastly, in Chapter 11, the author introduces some further useful constructions in algebraic topology, like cones and suspensions, and very briefly discusses the notions of fiber and vector bundle. This second part naturally requires some basic knowledge of group theory and linear algebra.

A list of exercises is provided at the end of each chapter and answers to some of them are given at the end of the book.

The book is remarkably free of typos and the few that I have found can be easily detected by a careful reader. Just to give an idea, for instance, the third line of Example 5.50 on page 79 should read "homeomorphic to S^2" instead of "homeomorphic to S^1"; the term tx in the formula for H of Example 6.14 on page 98 should appear multiplied by 2; the second example on page 169 should read "$\delta_3(\delta_4(f))$" instead of "$\delta_2(\delta_3(f))$"; etc.

In summary, this is a good, well-written textbook which is strongly recommended for beginning undergraduates. Those students who intend to carry out research in pure mathematics most likely will need to take more advanced courses in Topology. However, this book will offer them an excellent and very attractive overview of the entire field.

A Taste of Topology

by Volker Runde

Universitext. Springer-Verlag,
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reviewed by Ariel Blanco, Department of Pure Mathematics, School of Mathematics and Physics, Queen's University Belfast, Belfast BT7 1NN, a.blanco@qub.ac.uk

The task of preparing a good set of lecture notes is often time and energy consuming, especially, if you want to cover the most important results of a particular subject in a one-semester course. For this reason, if you are thinking of teaching a course on point-set topology then the book under review might be worth having a look at.

"A Taste of Topology", while being a short book (about 150 pages without counting appendices) covers in a serious, accessible and well organized manner the most important parts of the subject and even topics which are usually left out in courses at undergraduate level. Like nets, for instance. This makes it an excellent choice as a textbook for a one-semester course on set-theoretic topology.

Though, in principle, the book is self-contained enough as to be read by a diligent second year undergraduate, in my opinion, the way the material is presented, makes it more appropriate for final-year undergraduate or first-year postgraduate students who have been exposed to Analysis (real and complex) and Ring Theory.

The book starts off with a brief recalling of basic notions and results from set theory. This is done in Chapter 1. Then it continues with metric spaces in Chapter 2. Several topological concepts are first discussed in this setting, although reference to metrics is avoided in the proofs whenever possible. Baire's category theorem is derived from a simplified version of Bourbaki's Mittag–Leffler theorem. The chapter concludes with a discussion of compactness for metric spaces.

The next two chapters, Chapters 3 and 4, could perfectly be the core of a one-semester course on point-set topology.

In Chapter 3, the notion of a topological space is introduced explicitly. Here the author discusses in detail the most important ways of axiomatizing the concept. Among the examples discussed throughout the chapter figure the Zariski topology and different topologies on spaces of functions. A very gentle introduction to nets follows and a short intuitive proof of Tychonoff's Theorem using nets is given. Connectedness and separation properties are also discussed in this chapter.

Families of continuous functions is the topic of Chapter 4. The chapter starts with Urysohn's Lemma, Urysohn's metrization and Tietze's extension theorems. Then the author continues the study of compactifications initiated in Chapter 3, this time with the Stone–Čech compactification. The chapter ends with the Stone–Weierstrass theorem. An elegant proof of this last result is given, though in the reviewer's opinion, not a very transparent one, bearing in mind that the book is mainly intended for students.

Finally, Chapter 5 is an 'invitation' to algebraic topology. It touches very briefly the important concept of homotopy equivalence and defines the fundamental group and covering spaces of a topological space.

Each section is accompanied by a list of exercises, and each chapter ends with a very enjoyable section of historical remarks and suggestions for further reading.

Although, as mentioned by the author in the introduction, the book reflects his own preferences, I believe it still covers a significant amount of material. One minor criticism in this direction is the total omission of filters, even though, as the author, I also feel the notion of net, being closer to that of sequence, should be easier to grasp by the student.