where p may have singularities at $t = 0$ and/or 1, and f may be singular at $y = 0$, in that

$$f : [0,1] \times (0,\infty) \times (-\infty,\infty) \to (0,\infty)$$

is continuous, $\lim_{y\to 0^+} f(x,y,v) = +\infty$ uniformly on compact subsets of $[0,1] \times (-\infty,\infty)$, and

$$f(t,y,v) \le [g(y) + h(y)]k(v),$$

where g is continuous, positive and non-increasing, and $h \ge 0$, $k > 0$ are both continuous on $[0,\infty)$. The most interesting case for (14), (15) is when $b = 0$. This is addressed by considering the appropriate one-parameter homotopy family for (14) satisfying

$$y(0) = a > 0, \quad y(1) = \frac{a}{n}, \quad n \in \mathrm{N}, \tag{16}$$

making sufficient assumptions so that *a priori* bounds are obtained for this associated family of BVP's, again obtaining positive solutions $y_n(t)$, and then passing to the limit, with Arzela-Ascoli providing a positive solution of (14), (15), with $b = 0$.

This is a very readable and attractive book, containing much basic information and with a contemporary outlook on singular BVP's for second order ODE's. The references give an adequate sample of the relevant literature on this topic.

A PROBLEM OF BOURBAKI ON FIELD THEORY

Rod Gow

The following problem appeared in one of Bourbaki's early chapters on algebra, [1, p.146]. Let K be a commutative field of characteristic different from 2 and let f be a mapping of K into itself such that

$$f(x+y) = f(x) + f(y)$$

for all x and y in K and

$$f(x)f(x^{-1}) = 1$$

for all non-zero x. Show that f is an isomorphism of K onto a subfield of K (or alternatively, a monomorphism of K). In other words, we must show that

$$f(xy) = f(x)f(y)$$

for all x and y.

In fact, Bourbaki's result is not strictly true as it stands. For it follows from the relation $f(x)f(x^{-1}) = 1$ that $f(1)^2 = 1$ and thus $f(1) = \pm 1$. Now if $f(1) = -1$, f is not a monomorphism, but it can be proved that $-f$ defined by $(-f)(x) = -f(x)$ is a monomorphism. We will assume throughout this discussion that $f(1) = 1$. We note that Bourbaki's exercise was still being presented in the incorrect form in later editions such as [2, p.175].

A hint is given in Bourbaki's exercise: show that $f(x^2) = f(x)^2$ for all x (there is a misprint of this in [1]). It took us some time to prove the equality above and, to allow people to try to prove this for themselves, if they so wish, we will not present our proof (which is totally elementary). The proof does not require any restriction on the characteristic of K. Now, assuming that $f(x^2) = f(x)^2$, we can easily prove Bourbaki's result by considering the expansion of $f((x+y)^2)$.

We would like to raise two other questions. The first is: is the same result true when K has characteristic 2? We have checked that it is true for finite fields of characteristic 2. The other question we would like to mention is: what can be said if K is not necessarily commutative (that is, when K is a skew-field)? It is straightforward to see that the relation $f(x^2) = f(x)^2$ still holds in case K is a skew-field but the only general relation connecting x and y that we have been able to obtain is

$$f(xy(x+y)^{-1}) = f(x)f(y)(f(x)+f(y))^{-1},$$

which holds for all non-zero x and y, with $x \neq -y$, provided that x and y commute and $f(x)$ and $f(y)$ commute.

In conclusion, we suspect that it is quite likely that this question has already been discussed in the literature, although we have not seen anything ourselves.

References

[1] N. Bourbaki, Algèbre, Chapitre I-Structures algébriques (second edition). Hermann: Paris, 1951.

[2] N. Bourbaki, Algebra, Chapters 1-3. Springer-Verlag: Berlin-New York-Heidelberg, 1989.

Rod Gow
Department of Mathematics
University College
Belfield
Dublin 4
email: rodgow@irlearn.ucd.ie

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