Abstract of Doctoral Thesis

TOTAL NEGATION IN GENERAL TOPOLOGY AND IN ORDERED TOPOLOGICAL SPACES

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Total negation is a procedure, formulated by Paul Bankston [Illinois J. Math. **23** (1979), 241–252], whereby each topological invariant P gives rise to another denoted anti-P, a space being called anti-P when none of its subspaces is P except for those whose point-sets, by virtue of their cardinalities, cannot sustain a non-P topology. This thesis begins by surveying the previous literature on the topic, and consolidates it by investigating the total negations of several invariants (including separability, first and second countability and separation axioms weaker than T_1) and by establishing previously unnoticed implications amongst a number of conditions related to anti-compactness.

Then the theory is extended in three principal directions. One of these arises from Brian Scott's characterization of those invariants P for which an invariant Q can be found whose total negation is P; here the circumstances in which Q may be taken to be hereditary are identified, and several results are obtained about the class of all such hereditary properties Q for a given P, for example, concerning whether or not this class possesses a weakest or a strongest member.

Another is an exhaustive investigation of the possibilities occurring when the anti-operation is iterated, especially with

regard to the repetitive patterns of invariants thus generated; a sample conclusion is that for any P either every topological space is anti^3-P or else anti^2-P =anti^3-P [J. Inst. Math. & Comp. Sci. (1990), 31–35].

The third extension is a broadening of these ideas into other categorical settings, beginning with an account of total negation in the context of ordered topological spaces in which the rôle played by *cardinality* of subspaces in the definition of anti-P is replaced by *order-type*. Many of the classical notions carry over to this setting, including versions of the iteration theorem and of the Scott characterization. The final chapter provides a categorical perspective for this material, indicating the possibility of parallel theories in groups, dimension theory, pure partial order etc. [Boll. Unione Mat. Ital., to appear].

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