If $Z(G)$ is finite we are finished, so assume that $Z(G)$ is infinite. Then

$$Z(G) \simeq T \times C_\infty \times \cdots \times C_\infty$$

is the direct product of a finite group T and finitely many infinite cyclic groups. Now $Z(G)$ is invariant under all epimorphisms of G onto G, but clearly $\alpha : x \to x^{n+1}$ is not onto on any of the infinite cyclic factors. This contradiction establishes the result.

Reference

[1] S. V. Ivanov, *On the Burnside problem on periodic groups*, Bull. Amer. Math. Soc. **27** (1992), 257–260.

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THE POISSON KERNEL AS AN EXTREMAL FUNCTION

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It has long been known that for certain classical inequalities involving positive harmonic functions on the open unit ball B of $\mathbf{R}^N$ the Poisson kernel of B (with some fixed pole on ∂B) is extremal (that is, a function for which equality holds in the inequalities). In recent years several new inequalities for positive harmonic functions on B have been discovered; again the Poisson kernel and functions related to it appear in extremal roles. This article, which is based on part of a talk given at the Society's Meeting at Waterford in September 1992, surveys some such inequalities, both old and new.

1. Harmonic functions and the Poisson kernel

1.1. *Harmonic functions*

A real-valued function h is *harmonic* on a non-empty open subset Ω of the Euclidean space $\mathbf{R}^N$, where $N \geq 2$, if h is smooth (that is, $h \in C^2(\Omega)$) and satisfies Laplace's equation (that is, $\Delta h \equiv 0$ on Ω, where $\Delta = \partial^2/\partial x_1^2 + \ldots + \partial^2/\partial x_N^2$). Harmonic functions are also characterized by Gauss' mean value property: h is harmonic on Ω if and only if h is continuous on Ω and, for each closed ball $\beta \subset \Omega$, the value of h at the centre of β is equal to its average value over the boundary $\partial\beta$ of β (see, e.g., Hayman and Kennedy [13, §1.5.5]).

[10] A. E. McCluskey, Minimality with respect to separation axioms and sobriety, including their induced order characterizations, (Ph.D. thesis). Queen's University Belfast, 1990.

[11] A. E. McCluskey and S. D. McCartan, *Minimal sober spaces* (submitted).

[12] A. E. McCluskey and S. D. McCartan, *The minimal structures for* T_A, Ann. New York Acad. Sci. **659** (1992), 138–155.

[13] A. E. McCluskey and S. D. McCartan, *Minimality with respect to Young's axiom* (submitted).

[14] J. van Mill and G. M. Reed (eds.), Open problems in topology. North-Holland: Amsterdam-New York-Oxford-Tokyo, 1990.

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EPIMORPHISMS ACTING ON BURNSIDE

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The Burnside group $B(r,n)$ is the group of exponent n, generated by r elements $x_1, x_2, \ldots, x_r$. It is well known that $B(r,n)$ is finite for $n = 2$, 3, 4 and 6 for all r but that for $n \geq 665$ and n odd, $B(r,n)$ is infinite when $r > 1$. In addition, it has recently been shown that for $n \geq 2^{48}$, $B(r,n)$ is infinite for $r > 1$, [1].

Let $\mathcal{B}$ be the set of all positive integers n for which $B(r,n)$ is finite for all r. Since the relation $g^n = 1$ can be written as $g^{n+1} = g = (g)I$ where I is the identity automorphism, we ask the following question.

Suppose G is a finitely generated group and the map α given by $g\alpha = g^k$ for all $g \in G$ and a fixed positive integer k, is an automorphism of G. What values of k force G to be finite?

In fact, in what follows, we can replace 'automorphism' by 'epimorphism', that is, an endomorphism of G onto G, and prove the following result.

Theorem. *Suppose that n belongs to $\mathcal{B}$ and that G is a finitely generated group such that the map α given by $g\alpha = g^{n+1}$ for all $g \in G$ is an epimorphism of G. Then G is finite.*

Proof: For all a and b in G, $(ab)\alpha = (ab)^{n+1} = a^{n+1}b^{n+1}$, so by cancellation $(ba)^n = a^n b^n$. Then $(ba)^{n+1} = (ba)^n ba = a^n b^n ba$, whence $b^{n+1}a^n = a^n b^{n+1}$. Since α is onto, $ga^n = a^n g$ for all a and g in G, and so $a^n \in Z(G)$ for all $a \in G$, where $Z(G)$ denotes the centre of G.

Now $G/Z(G)$, being a factor group of a finitely generated group, is finitely generated of exponent n and since $n \in \mathcal{B}$, $G/Z(G)$ is finite. Thus $Z(G)$, being a subgroup of finite index in a finitely generated group, is a finitely generated abelian group.