Research Announcement

THE τ_w TOPOLOGY ON SPACES OF HOLOMORPHIC FUNCTIONS

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If U is a domain in a locally convex space over C then a seminorm p on $H(U)$, the space of complex valued holomorphic functions on U, is said to be τ_w-continuous if there exists a compact subset K of U such that for each V open, $K \subset V \subset U$, there exists $C(V) > 0$ such that

$$p(f) \leq C(V)\|f\|_V$$

for $f \in H(U)$.

The τ_w topology is the topology generated by all τ_w continuous seminorms. The τ_w topology was originally motivated by properties of analytic functions which can be represented by Borel measures supported by every neighbourhood of compact set but not by the compact set itself and at the linear level is related to the inductive dual of a locally convex space.

The τ_w topology is thus defined by a set of inequalities and in many cases it is of interest to find an explicit set of semi-norms which generated this topology. Explicit sets are known for balanced domains in Banach spaces and in Fréchet-Montel spaces where it is known that τ_w coincides with the compact open topology. Here we give an explicit set of semi-norms for a collection of Fréchet spaces which includes all Banach spaces, all Fréchet spaces with unconditional basis of type (T) and the Köthe echelon spaces.

Definition 1. *An unconditional Schauder decomposition, $\{E_n\}_n$ of a Fréchet space E is a T-Schauder decomposition if there exists a fundamental system of semi-norms for E, $(\|\cdot\|_k)_{k\in N}$ such that*

(i) $\|P_J(x)\|_k \leq \|x\|_k$ *all $J \subset N$, $k \in N$ and $x \in E$*

(ii) for every sequence $\alpha = (\alpha_k)_k$, $0 < \alpha_k \leq 1$, there exists a partition $J_\alpha = (J_{\alpha,k})_k$ of N such that if $P_{\alpha,k} := P_{J_{\alpha,k}}$ then $\|P_{\alpha,k}(x)\|_{k-1} \leq \alpha_k \|P_{\alpha,k}(x)\|_k$ for all $x \in E$ and all $k \geq 2$.
(iii) $(\|\cdot\|_k)$ defines the topology induced by E on $P_{\alpha,k}(E)$ for all α and all k.

Fréchet spaces with a T-Schauder decomposition are a slight modification of the spaces introduced in [1] and the spaces in [1] appeared as a result of developments arising form positive solutions to Grothendieck's "Problème des topologies".

Theorem 2. *If the Fréchet spaces E has a T-Schauder decomposition then the τ_w topology on $H(E)$ is generated by all semi-norms of the form*

$$p(f) = \sum_{n=0}^{\infty} \left\| \frac{\hat{d}^n f(0)}{n!} \right\|_{B_n}$$

where $(B_n)_n$ is a sequence of compact subsets of E which converges to a compact subset of E.

We have written $\sum_{n=0}^{\infty} \frac{\hat{d}^n f(0)}{n!}$ as the Taylor series expansion of f at the origin.

Reference

[1] J. Bonet and J. C. Diaz, *The Problem of topologies of Grothendieck and the class of Fréchet T-spaces*, Math. Nachr. **150** (1991), 109–118.

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