F is algebraically closed and $\mathcal{A}$ is indecomposable holds the key concepts. If A, say, has r Jordan blocks, with the biggest Jordan block of size $k \times k$, then it is shown that generally $\dim_F \mathcal{A} \leq \{nk, kr(r+1)/2\}$. In the homogeneous case, it is shown that $\dim_F \mathcal{A} \leq n^{3/2}$, and if $\mathcal{A}$ has fewer than four Jordan blocks, then $\dim_F \mathcal{A} \leq n$. Further if the exponent of the algebra $\mathcal{A}$ is also k (i.e. $\mathcal{A}^k = 0$), then it is shown that for $n < 30$, $\dim_F \mathcal{A} \leq n$. In case $\mathcal{A}$ is homogeneous, then each matrix in $\mathcal{A}$ can be considered as an element of $M_r(F[J])$ (where $A = J \oplus \cdots \oplus J$, r blocks of $J = J_k$, the $k \times k$ Jordan block with associated eigenvalue zero). It is shown that if B is a Wasow matrix over the local commutative ring $F[J]$, i.e., B is similar over $F[J]$ to a matrix in rational canonical form, then again in this case the dimension of $\mathcal{A}$ cannot exceed n.

Let $\mathcal{A}$ be a commutative subalgebra of $M_n(F)$, and say the centralizer of $\mathcal{A}$, $\mathcal{C}(\mathcal{A})$, is contained in $\mathcal{A}$. Then $\mathcal{A}$ is said to be a maximal commutative subalgebra of $M_n(F)$. We define the exponent of $\mathcal{A}$ to be the smallest positive integer k such that $x_1 \ldots x_k = 0$ for all $x_1, \ldots x_k$ in the radical of $\mathcal{A}$. In Chapter III we study the dimensions of maximal commutative subalgebras of $M_n(F)$. A classical result of Schur states that $\dim_F \mathcal{A} \leq [1 + n^2/4]$, where [ ] denotes the greatest integer function. Courter [Duke Math. J. 32:225-232 (1965)] proved if $\mathcal{A}$ has exponent two then $\dim_F \mathcal{A} \geq n$. Laffey [Linear Alg. Appl. 71:199-212 (1985)] showed that generally $\dim_F \mathcal{A} \leq (2n)^{2/3} - 1$, and if $\mathcal{A}$ has exponent three then the best possible lower bound is $[3n^{2/3} - 4]$. We create a sequence of maximal commutative subalgebras $\mathcal{A}_n$, each with exponent four, with $\dim_F \mathcal{A}_n$ of the order of $n^{2/3} - n^{1/3}$, in the limit. On the other hand, if the exponent of $\mathcal{A}$ is greater than or equal to $n-3$, and the characteristic of F does not divide $n!$, then we show that $\dim_F \mathcal{A}$ is either n, $n+1$ or $n+2$.

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Research Announcement

PREDUALS OF SPACES OF HOLOMORPHIC FUNCTIONS

Christopher Boyd

For U an open subset of a locally convex space E we denote by $\mathcal{H}(U)$ the space of $\mathbb{C}$–valued holomorphic functions on U. In infinite dimensional holomorphy we consider three natural topologies on $\mathcal{H}(U)$. τ_o is the compact–open topology of convergence on compact subset of U. We say a semi–norm p is ported by the compact subset K of U if for each open set V, $K \subset V \subset U$, we can find $c(V) > 0$ such that $p(f) \leq c(V)\|f\|_V$ for every f in $\mathcal{H}(U)$. τ_ω is the topology generated by all semi–norms ported by compact subsets of U. Finally say that a semi–norm p is τ_δ continuous if for each countable increasing open cover $\{U_n\}_n$ of U there is $C > 0$ and $n_o \in \mathbb{N}$ such that $p(f) \leq C\|f\|_{U_{n_o}}$ for every $f \in \mathcal{H}(U)$. τ_δ is the topology on $\mathcal{H}(U)$ generated by all τ_δ continuous semi–norms. We always have

$$\tau_o \leq \tau_\omega \leq \tau_\delta$$

on $\mathcal{H}(U)$. $P(^nE)$ denotes the space of n-homogeneous polynomials on E. We note that τ_ω and τ_δ agree on $P(^nE)$ for every integer n. For K a compact subset of E we let $\mathcal{H}(K)$ denote the space of holomorphic germs on K. The τ_o(resp. τ_ω) topology on $\mathcal{H}(K)$ is defined by $(\mathcal{H}(K), \tau_o) = \mathrm{ind}_{K \subset V}(\mathcal{H}(V), \tau_o)$ (resp. $(\mathcal{H}(K), \tau_\omega) = \mathrm{ind}_{K \subset V}(\mathcal{H}(V), \tau_\omega)$).

Given a locally convex space E we let $E'_i = \mathrm{ind}_V E'_{V^\circ}$, where the inductive limit is taken over all neighbourhoods V of 0 in E, and let E'_b denote the dual of E equipped with the topology of uniform convergence on bounded subsets of E.

In [3] Mujica and Nachbin shows there is a complete locally convex space $G(U)$ with the property that $G(U)'_i = (\mathcal{H}(U), \tau_\delta)$.

By construction

$$(\mathcal{H}(U), \tau_o)'_b \subseteq G(U) \subseteq (\mathcal{H}(U), \tau_\delta)'_b.$$

We consider a special case, when U is a balanced open subset of a Fréchet (complete metrizable) space E, and show how the topological properties of $G(U)$ are related to the topological properties of E. In this situation we note that $G(U)$ is neither metrizable nor dual metrizable except in the trivial case when E is finite dimensional.

Reflexivity

Firstly we consider the question: When is $G(U) = (\mathcal{H}(U), \tau_\delta)'_b$? The following theorem characterizes U with this property.

Theorem 1. *For a Fréchet space E the following are equivalent:*
(i) $(P(^nE), \tau_o)' = (P(^nE), \tau_\omega)'$ *for every integer n.*
(ii) $(\mathcal{H}(U), \tau_o)' = (\mathcal{H}(U), \tau_\omega)'$ *(resp.* $(\mathcal{H}(K), \tau_o)' = (\mathcal{H}(K), \tau_\omega)'$*) for one and hence every balanced open (resp. compact) subset U (resp. K) of E.*
(iii) $(P(^nE), \tau_\omega)$ *is reflexive for every integer n.*
(iv) $(\mathcal{H}(U), \tau_\omega)$ *is semi-reflexive for one and hence every balanced open subset U of E.*
(v) $(\mathcal{H}(U), \tau_\delta)$ *(resp.* $\mathcal{H}(K), \tau_\omega)$*) is reflexive for one and hence every balanced open (resp. compact) subset U (resp. K) of E.*
(vi) $G(U) = (\mathcal{H}(U), \tau_\delta)'_b$ *for one and hence every balanced open subset U of E.*
(vii) $(\mathcal{H}(U), \tau_\omega)'_b \subseteq G(U)$ *for one and hence every balanced subset U of E.*

Alencar, Aron and Dineen showed in [1] that $(\mathcal{H}(U), \tau_\delta)$ is reflexive for every balanced open subset of Tsirelson's space. Therefore there are Banach spaces with unconditional basis for which $G(U) = (\mathcal{H}(U), \tau_\delta)'_b$. The above Theorem and a result of Aron (see [4]) also allow us to show that $G(U) \neq (\mathcal{H}(U), \tau_\delta)'_b$ when U is a balanced open subset for any ℓ_p space.

Distinguishedness

Although $G(U)$ may not be equal to $(\mathcal{H}(U), \tau_\delta)'_b$ it still may be possible that the inductive dual of $G(U)$, $G(U)'_i$ is equal to the strong dual of $G(U)$, $G(U)'_b$. When this occurs we have $(\mathcal{H}(U), \tau_\delta) = G(U)'_b$. To see when it does happen we need the notion of distinguishedness.

A locally convex space E is said to be distinguished if every $\sigma(E'', E')$-bounded set of E'' is contained in the $\sigma(E'', E')$-closure of some bounded set of E. If E is a Fréchet space then E is distinguished if and only if $E'_i = E'_b$. When U is a balanced open subset of a Fréchet space, $G(U)$ satisfies the following analogous theorem.

Theorem 2. *Let U be a balanced open subset of a Fréchet space E, then $G(U)$ is distinguished if and only if $G(U)'_i = G(U)'_b$.*

It can be shown that $G(U)$ is distinguished for every balanced open subset of a Banach space with an unconditional basis. In particular this will mean that $G(U)'_b = (\mathcal{H}(U), \tau_\delta)$ for every balanced open subset of ℓ_p, $0 \leq p < \infty$.

Quasinormability

Grothendieck [2] introduced the notion of quasinormability as a property which was stable under a large number of topological vector space operations (e.g. formation of biduals, projective tensor products, etc).

A locally convex space E is said to be quasinormable if for each absolutely convex neighbourhood U of 0 in E there is an absolutely convex neighbourhood of 0, V, such that given $\alpha > 0$ there is a bounded subset B_α of E with

$$V \subset B_\alpha + \alpha U.$$

Every normed space is quasinormable, while a Fréchet–Montel space is quasinormable if and only if it is Schwartz. We have the following theorem concerning the quasinormability of $G(U)$.

Theorem 2. *Let U a balanced open subset of a Fréchet space E, then $G(U)$ is quasinormable if and only if E is quasinormable.*

These results are part of the authors doctoral thesis under Prof. S. Dineen, which is due to be submitted to the NUI in 1992.

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PROPOSED SOLUTION TO A PROBLEM OF MINIMAL DIRECTION IN TYPESETTING

Notes for contributors

Mícheál Ó Searcóid

Abstract: This is a first attempt to present to contributors of the Bulletin a plain TEX package which will allow them to do most of their own editing and to present papers in a form more or less ready for publication. The article also describes how to use *MISTRESS*, a system for typesetting references in many journals with different styles.

Introduction

The Bulletin has been beset by production problems over the last couple of years. Rex Dark and I, as acting editor and production manager, decided that we should make a concerted effort to tackle these problems in a way which would both enable us to get the Bulletin back on schedule by early 1993 and make it easier for future editors to get issues out on time. In this article, I should like firstly to make some observations and suggestions to the mutual benefit, I hope, both of the production team of the Bulletin and of its contributors. Secondly, I will describe the solution to the problems which arose from those observations.

Submissions to the Bulletin

It was decided at a recent meeting of the IMS that contributors should be encouraged to submit articles in TEX if they have the facilities to do so. Since our typesetting is done in TEX, it is reasonable to suppose that TEX-written papers will be processed more quickly than those that are not so written; indeed some journals