BOOK REVIEWS

INVERTIBILITY AND SINGULARITY FOR BOUNDED LINEAR OPERATIONS by Robin Harte
Marcel Dekker, 1987, 528 pp, $119.50, ISBN 0-8247-7754-9

Reading the preface I was struck by the thought that this book would be something of a journey into the author's mind. The prospect filled me with some trepidation. In fact the journey was both better and worse than I had expected. Let me explain. Early in the preface the author states

> We have tried to write an introduction to operator theory accessible to students meeting the definitions for the first time.

Good, I thought. A little later I met

> The reader will also probably find our obsession with incomplete spaces tedious.

This sounded warning bells. In fact it transpired that the word accessible is given a breath of understanding not usually accorded to the word. Of this the author is clearly aware, and hence the honesty of the second statement.

This book really contains two books. One of these is a compendium of the spectral theory (in all shapes and sizes) of linear operators on Banach spaces. The second is an exploration of what happens if the space is simply a normed linear space and not necessarily complete. If the reader's interest is in the first book, then tedious is probably too mild a word to use. However, if one has a thorough understanding of the material of the first book, then the second book has quite a lot of interest. But not as part of "an introduction to operator theory". Intertwining the two books means that the entire is not, as a single entity, suitable for learning the subject. There are other drawbacks. In the first five chapters the basic results of elementary Banach space theory are covered. These include the big three, the open mapping, the uniform boundedness and the Hahn-Banach theorems. As in most of the book it is clear that the author has put a lot of thought into the means of presentation, with creditable results. However there is also a lot of less

important material interspersed, much of it from the second book. There is not sufficient discrimination between the really basic material and that which would be consigned to the exercises and notes section of other texts. This evenhanded treatment persists throughout the book so that the multitude of notions and notations lacks a hierarchy of importance.

Speaking of notations, there are three and a half pages of special symbols at the end of the book. And even this is not enough. On page 40, one encounters "Strictly speaking, we should write something like $l_p(\Omega, T)$ for the restriction of the mapping T^Ω: we shall however continue to write T^Ω." Then three lines later one meets "$l_p(\Omega, T)$" and two lines later "$C_\infty(\Omega, T)$", at whose meaning one must guess, as it is not among the aforementioned three and a half pages.

Additionally, it is confusing to have $x \in X$ on one line and $x \in X^\Omega$, a function space, two lines later. Items such as these and the many typographical errors make the technical material more difficult. In the preface the author admits to many "silly mistakes—left as a series of unspoken exercises for the reader". Unfortunately silly mistakes make silly exercises.

Another example of the lack of a firm editor is the use of the term Hilbert Algebra, confessedly "in total defiance of standard terminology" to describe a C^*-algebra, whilst even acknowledging the fact that Hilbert algebra is already in general use for something else.

In general the book seeks to illuminate even slight nuances of difference and to treat things in great generality. Thus sequences are not confined to being indexed by the natural numbers **N**, but are indexed by any bornological space Ω (Chap 1, §8), to what gain is not clear. And yet in other places distinctions are blurred. "We can treat a polynomial as a mapping" (page 21).

The pity is that these and other instances distract the reader from the many fine sections, for example the canonical factorisation of Theorem 2.3.3.

I have commented on chapters 1–5 which are "Normed Linear Spaces", "Bounded Linear Operations", "Invertibility and Singularity", "Banach Spaces and Completeness", and "Linear Functionals and Duality". I should mention also that the reader is introduced here to "Enlargements" §1.9, 2.7, 5.7, which allow one to replace statements about an operator on a space X, with statements about an "enlarged" operator on an "enlarged" space, and to "Composition Operations" in §2.9, and §5.6. These also recur throughout the book but one is left to wonder about their importance to the subject.

Chapter 6 is "Finite Dimensional Spaces and Compactness". This runs from defining linear independence to almost upper/lower semi-Fredholm. It includes a nice treatment of the equivalence of norms on finite dimensional spaces.

Chapter 7 is "Operator Algebra and Commutativity". This contains a lot of basic functional analysis, for example the "Stone-Weierstrass Theorem". Chapter 8 is "Inner Products and Orthogonality". The title says it all— again basic functional analysis. Chapter 9 is "Liouville's Theorem and Spectral Theory". Another good basic chapter, including the beginnings of the theory of C^*-algebras and their representations.

Chapter 10 is "Comparison of Operators and Exactness". The introduction to this chapter states "The various kinds of invertibility have "relative" analogues, in which one operator is compared to another. If we mix both left and right comparisons and then specialize we come down to concepts of "exactness"". Enough said.

Chapter 11 is "Multiparameter Spectral Theory". This contains the Taylor spectrum, an idea toward which much of the book seems aimed. There is also useful material on the Silov boundary and Tensor Products.

A final section is a collection of "Notes, Comments, and Exercises" for each chapter. It is clear from these that the author has researched his subject with diligence and thoroughness, and this section adds greatly to the value of the book as a compendium of results. In fact one is tempted to suggest that the book might have been called "everything you ever wanted to know about Spectral Theory, but were afraid to ask—in case you were told". The one thing that could be found missing is some mention of the many areas in which Spectral Theory finds its applications, and which provide it's Raison d'Etre.

Much of the above comment may seem negative, so let me hasten to add that this is a book that I am glad to have on my shelves. It has appeal on three levels. Firstly, the standard introductory results of functional analysis and operator theory are all there. Secondly, it collects many of the more esoteric notions of Spectral Theory, and finally it contains the authors own ruminations on completeness and the lack of it.

Donal O'Donovan
School of Mathematics
Trinity College Dublin.

PROBLEM PAGE

Editor: Phil Rippon

Here are a couple of attractive problems which fall into the category of 'geometric doodling'. The first one appears in Coxeter's book 'An Introduction to Geometry', but I heard it first from my school maths teacher.

21.1 *What is the minimum number of (strictly) acute angled triangles into which a square can be partitioned?*

The next problem was asked recently by an OU maths student at summer school. It has a very neat solution and I'd be interested to hear of any references to it.

21.2 *Find a configuration of finitely many points in the plane such that the perpendicular bisector of each pair of the points passes through at least two of the points.*

Finally, a wonderful sequence problem due to John Conway, who offered a prize of \$1000 in July this year for a solution (his audience thought he had offered \$10,000!).

21.3 *Let $a(n)$, $n = 1, 2, \ldots$, be defined as follows: $a(1) = 1$, $a(2) = 1$, and*

$$a(n+1) = a(a(n)) + a(n+1-a(n)), \qquad n = 2, 3, 4, \ldots$$

Thus the sequence begins:

$$1, 1, 2, 2, 3, 4, 4, 4, \cdots.$$

The problem is to determine an integer N such that

$$\left|\frac{a(n)}{n} - \frac{1}{2}\right| < 0.05 \qquad \text{for } n > N.$$

A solution was given three weeks later by Colin Mallow, a mathematician working at Bell Labs. In September, a British newspaper offered a magnum