difficult and in many cases draw on obscure (to me at any rate) results in functional analysis. Bedside reading it is NOT!

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"PARTIAL DIFFERENTIAL EQUATIONS FOR SCIENTISTS AND ENGINEERS"

By *G. Stephenson*

Published by the *Longman Group Ltd*, 1984. x + 161 pp.
ISBN 0-582-44696-1.

Dr Stephenson, who is the author of several textbooks on mathematics, has written a compact and eminently readable account of partial differential equations at an elementary level. The book itself is intended for scientists and engineers, and the inclusion of the last name in the title is evidence of the increasing sophistication of the mathematical techniques required of the engineer nowadays.

By normal standards Dr Stephenson's book is small, yet he has succeeded in including a large amount of useful information within its 160 pages. He concentrates mainly upon the so-called equations of mathematical physics, e.g. Laplace's equation, the wave equation, the heat conduction equation and Schrodinger's equation. Occasionally functions involving more than two independent variables are considered. In discussing boundary value problems the author inserts a short section on ill-posed problems.

The book commences with a classification of the second order partial differential equations. Then orthonormal functions are introduced, with a brief but welcome reference to completeness, after which the author moves on to the question of the separation of variables, which could probably be described as the heart of the book. Numerous well-chosen examples of this useful technique are included, while at the same time the reader is introduced to the notion of discrete eigenvalues and eigenfunctions. Sturm-Liouville theory is mentioned, admittedly only briefly, but some substantial results are obtained. Solving Laplace's equation in three dimensions in non-rectangular coordinates gives rise naturally to the so-called special functions, but, as the author is careful to state in his preface, little space is devoted to them and any of their properties which are used are stated without proof. There is also a short but useful chapter on continuous eigenvalues and Fourier integrals.

In a book of this size a chapter on the Laplace transform might be regarded as a luxury, but the author's decision to include one is undoubtedly justified. There are many excellent examples of the application on the Laplace transform, and indeed of the Fourier transform, to boundary value problems involving partial differential equations. There is also an introduction to the Green's function, the Heaviside step function and the delta function.

It is hardly possible nowadays to write a textbook on partial differential equations without some reference to numerical methods and the tremendous impetus given to these methods by the recent explosion in computer technology. One can only agree wholeheartedly with the author's own cogent observation that results which are spurious may be accepted as correct just because they come out of a computer.. Perhaps from a desire to combat this dangerous tendency, the author devotes his last chapter to a brief exposition, mostly by way of examples, of the finite difference and the finite element methods. The Rayleigh-Ritz idea, which relies heavily upon the calculus of variations, is used to illustrate the finite element approach.

Every chapter is followed by a set of problems and answers to all the problems are provided. The book is remarkably devoid of misprints, in fact the only nontrivial one I encountered occurs in the second of Problems 5, page 77, which deals with the generating function for $J_n(x)$. I feel that the author has succeeded admirably in his intention of producing an elementary text which is accessible to any undergraduate student with fairly basic mathematical education and I should have no hesitation in using this book should the occasion arise.

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"POLYHEDRAL COMBINATORICS AND THE ACYCLIC SUBDIGRAPH PROBLEM"

By *M. Junger* (Research and Exposition in Mathematics 7)

Published by *Heldermann Verlag*, Berlin, 1985. x + 128 pp. DM 36. ISBN 3-88538-207-5.

Combinatorial optimisation [1,2] is the branch of mathematics which tackles such problems as the travelling salesman problem, shortest path problems, matching problems and network flow problems. More specifically, if S is a non-empty finite set and f is a real-valued function on the subsets of S then combinatorial optimisation refers to the problem of maximising f on a given collection of subsets. Since S is finite the most obvious way of solving such a problem is to list all the values of f in question and to pick the largest one. This method is too naive to be of much practical use. Instead, such problems are solved by developing algorithms for finding the required solution. Combinatorial optimisation is a child of the computer age. Not only are computers used to find the solutions, but a number of the problems in the field have arisen

in research in computer design and the theory of computation. There are many "real-world" problems which can be solved by applying the techniques of combinatorial optimisation. (With regard to applications of optimisation techniques in the real world it is a salutory exercise to read the case studies in [3], e.g. "the celebrated brand X washing machine shipping catastrophe", which show how careful one must be in making decisions based on a mathematical model.)

The monograph under review discusses the following combinatorial optimisation problem: given a directed graph D with an integer "weight" on each arc, determine an acyclic subdigraph of maximum weight. An equivalent version of this *Acyclic Subdigraph Problem* (ASP) is the *Triangulation Problem*: find a simultaneous permutation of the rows and columns of a non-negative square matrix such that the sum of the entries above the diagonal of the permuted matrix is maximum. The Triangulation Problem has an application in economics.

Let the digraph D have n arcs. Each subset B of the arcs has a 0-1 n-dimensional incidence vector x_B associated with it. The acyclic subdigraph polytope $P_{AC}(D)$ is the convex hull of all x_B, where B runs over all acyclic arc sets in D. The ASP may then be formulated as the integer programming problem: maximise $c^t x$ subject to $x \in P_{AC}(D)$, given the non-negative vector $c \in Z^n$. The ASP is an example of an NP-hard problem (see [4]) and the idea of associating a polytope with the feasible set of such a problem and then of applying linear programming techniques, has become popular in recent times. It turns out to be crucial to determine the facets ((n-1)-dimensional faces) of the polytope, and the central achievement of this monograph is the determination of several classes of facets of $P_{AC}(D)$. The author expresses the confident hope that "the algorithmic exploitation of our results will in fact lead to the effective solution of large instances of real-world problems which can be formulated as an Acyclic Subdigraph Problem".