"PARAMETER ESTIMATION FOR STOCHASTIC PROCESSES"

By *Yu A. Kutoyants*

Published by *Heldermann Verlag*, Berlin, 1984. viii + 206 pp. DM 56. ISBN 3-88538-206-7.

This book is a translation by B.L.S. Prakasa Rao of a completely revised and extended version of a former book of Kutoyants, published in Russian. It has 200 pages divided into five chapters.

Chapter One consists of a quick review of the theory of parameter estimation when the data consist of independent observations. It briefly describes maximum likelihood estimation, Bayesian estimation, the Cramer-Rao lower bound on the mean squared error of an estimator and the less widely known theorem of Hajek establishing the asymptotic minimax lower bound for the risk of an estimator. The treatment here is discursive and clearly assumes a good degree of prior knowledge on the part of the reader.

Chapters Two through Four deal with three different situations where dependent observations arise and where the observations themselves consist of the values of a random function observed at all time points in an interval $[0,T]$. Chapter Two considers observations of the form

$$X(t) = S(\theta,t) + n(t), \quad 0 \leq t \leq T$$

where for each $\theta$, $S(\theta,.)$ is a known function of t, $(n(t) : 0 \leq t \leq T)$ is a gaussian process with mean zero and known covariance function and $\theta$ is the parameter to be estimated. Chapter Three considers observations on a diffusion process

$$dX(t) = S(\theta;t,X)dt + \sigma dW(t)$$



where for each $\theta$, $S(\theta;.,.)$ is a known function, $\sigma$ is a diffusion coefficient, $W(.)$ is the Wiener process and again $\theta$ is the parameter to be estimated. The differentials above are defined with respect to the Ito integral. Chapter Four considers observations on a Poisson process $(X(t) : 0 \leq t \leq T)$ with intensity function $S(\theta;.)$ where, for each $\theta$, $S(\theta;.)$ is a known function of t and it is required to estimate $\theta$. The three chapters are similarly organised beginning with the appropriate generalisations of the Cramer-Rao lower bound and Hajek's theorem and continuing on to investigate the consistency, efficiency and asymptotic mean squared error of the maximum likelihood and Bayesian estimates of $\theta$. Given certain smoothness conditions on the function S it is shown that both the maximum likelihood estimate and the Bayes estimate are asymptotically efficient. However in the absence of these smoothness conditions the maximum likelihood estimate and the Bayes estimate have different limiting properties and the Bayes estimate is the only asymptotically efficient estimate, This makes for an interesting divergence from the asymptotic equivalence of maximum likelihood and Bayes estimates monotonously encountered when dealing with independent and identically distributed observations. Examples are given of the applications of these results to situations that arise in signal processing and communications theory.

Chapter Five gathers together several general theorems on the properties of the likelihood ratio-theorems that have been used earlier in the proofs of Chapters 2-4. This is done since many of the proofs are identical or analogous for each of the observation types considered.

I am unable to judge the usefulness of this book from the point of view of an expert in this area. However, for someone familiar with estimation based on independent observations, it offers a clear insight into the difficulties involved in extending some of the results to the case of dependent observations. Perhaps unavoidably, the notation is complex, the proofs are difficult and in many cases draw on obscure (to me at any rate) results in functional analysis. Bedside reading it is NOT!

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"PARTIAL DIFFERENTIAL EQUATIONS FOR SCIENTISTS AND ENGINEERS"

By *G. Stephenson*

Published by the *Longman Group Ltd*, 1984. x + 161 pp.
ISBN 0-582-44696-1.

Dr Stephenson, who is the author of several textbooks on mathematics, has written a compact and eminently readable account of partial differential equations at an elementary level. The book itself is intended for scientists and engineers, and the inclusion of the last name in the title is evidence of the increasing sophistication of the mathematical techniques required of the engineer nowadays.

By normal standards Dr Stephenson's book is small, yet he has succeeded in including a large amount of useful information within its 160 pages. He concentrates mainly upon the so-called equations of mathematical physics, e.g. Laplace's equation, the wave equation, the heat conduction equation and Schrodinger's equation. Occasionally functions involving more than two independent variables are considered. In discussing boundary value problems the author inserts a short section on ill-posed problems.

The book commences with a classification of the second order partial differential equations. Then orthonormal functions are introduced, with a brief but welcome reference to completeness, after which the author moves on to the question of the separation of variables, which could probably be described as the heart of the book. Numerous well-chosen examples of this useful technique are included, while at the same time the reader is introduced to the notion of discrete eigenvalues and eigenfunctions. Sturm-Liouville theory is mentioned, admittedly only briefly, but some substantial results are obtained. Solving Laplace's equation in three dimensions in non-rectangular coordinates gives rise naturally to the so-called special functions, but, as the author is careful to state in his preface, little space is devoted to them and any of their properties which are used are stated without proof. There is also a short but useful chapter on continuous eigenvalues and Fourier integrals.



In a book of this size a chapter on the Laplace transform might be regarded as a luxury, but the author's decision to include one is undoubtedly justified. There are many excellent examples of the application on the Laplace transform, and indeed of the Fourier transform, to boundary value problems involving partial differential equations. There is also an introduction to the Green's function, the Heaviside step function and the delta function.

It is hardly possible nowadays to write a textbook on partial differential equations without some reference to numerical methods and the tremendous impetus given to these methods by the recent explosion in computer technology. One can only agree wholeheartedly with the author's own cogent observation that results which are spurious may be accepted as correct just because they come out of a computer.. Perhaps from a desire to combat this dangerous tendency, the author devotes his last chapter to a brief exposition, mostly by way of examples, of the finite difference and the finite element methods. The Rayleigh-Ritz idea, which relies heavily upon the calculus of variations, is used to illustrate the finite element approach.