application for the professorship in Cork (see [1]). Smith has done a very fine job in painstakingly deciphering the handwriting of both Boole and De Morgan, a difficult job at the best of times. He comments with great depth and perception on both the mathematical and personal content of each letter and in particular he examines very closely the trains of thought of both men in the crucial period 1847-1850 while symbolic logic was taking shape in their minds, albeit in different forms. To my mind, Smith has done a fine job and his book is indispensible to those interested in either Boole or De Morgan or indeed the history of mathematics in general. I can recommend the book very strongly and it should find a place in every University Library so that students can see the actual evolution of mathematical concepts.

Much as I would like to give a book such as this unqualified praise, I must draw attention to the number of misprints and elementary errors it contains. These are all the more surprising when one realises that the book has emanated from Oxford University Press, but thankfully there is nothing that a careful proof-reading of a second edition could not remedy. The following is a list of potential corrections:

1. Page 2 Boole was married in 1855 not 1856.
2. Page 3 "obituary" is mispelled.
3. Page 33 Archbishop MacHale's first name was John not William.
4. Page 38 "be" should be "by".
5. Page 40 The title of Boole's major work was "An Investigation of the Laws of Thought", not "An Investigation into the Laws of Thought".
6. Page 142 "Edition" is mispelled.
7. Page 148 "Cambridge" is mispelled.

The bibliography of Boole's printed works contains over twenty errors and slips, all of them minor, which I have attempted to correct in [1].

The misprints and errors to which I have referred detract only slightly from the book which I regard as a fine piece of scholarship and welcome warmly. Rumour has it that the author is at present working on a companion volume on the correspondence of Boole and William Thomson, Lord Kelvin. I look forward eagerly to its publication.

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"THE ONE-DIMENSIONAL HEAT EQUATION"

(Encyclopaedia of Mathematics and its Applications - Vol. 23, Section : Analysis)

By *John Rozier Cannon*

Published by *Addison-Wesley Publishing Co.*, 1984, xxiii + 483 pp., Stg £61.20. ISBN 0-201-13522-1

A few weeks ago, a colleague presented the following problem to a number of Applied Mathematicians, including myself:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + f ; \qquad 0 < x < 1, \qquad t > 0$$

$$u(x,0) = 0, \qquad 0 < x < 1$$

$$u(0,t) = u(1,t) = 0, \qquad t > 0$$

Are there any simple conditions on f that ensure the existence of a solution for which u, $\partial u/\partial x$, $\partial u/\partial t$ and $\partial^2 u/\partial x^2$ are continuous in $[0,1] \times [0,t]$?

This is a question of regularity and such problems are usually tricky. However, it relates to the one-dimensional heat equation and what could be simpler? Many helpful suggestions were offered but nobody knew the answer. I set about my task of reviewing Cannon's book with added interest. I was not disappointed. The solution to the above and many more complex problems may be found there. The main thrust of the book is towards problems of existence, uniqueness, stability and other properties of solutions. It consists of a blend of the research of Cannon and others with the classical material and is written in the form of a monograph.

An undergraduate background in real and complex analysis, Lebesgue integration, Fourier series and transforms, familiarity with the concept of a Banach space and an elementary course in partial differential equations would be more than adequate to enable a comfortable reading of the book. This is noteworthy since most modern monographs which address the type of material in Cannon's book require considerably more mathematical maturity on the part of the reader.

Cannon starts by collecting together a number of basic inequalities and results from real and complex analysis and Lebesgue integration for reference purposes in the preliminary Chapter 0. He then develops in Chapter 1, the Weak Maximum, Comparison and Uniqueness theorems (for solutions continuous in the closure of the space-time domain in which the differential equation holds - the *parabolic domain*) and states without proof the Extended Comparison and Uniqueness theorems for solutions which admit a finite number of discontinuities on the boundary of the parabolic domain (*the parabolic boundary*). The latter theorems are fundamental to the development of the subject matter (mainly for uniqueness proofs).

The pure initial-value problem and initial-boundary value problems are analyzed in Chapters 3 through 6. He starts with the fundamental solution and develops boundary-integral representations over the parabolic boundary of the solutions which provide the basis for existence/uniqueness proofs for quite general data. In Chapters 6 and 7 he reduces a wide range of initial-boundary value problems to the solution of systems of boundary-integral equations and in Chapter 8 makes use of this formulation to deduce existence/uniqueness, continuous dependence on the data and *a priori* bounds. Chapter 9 covers pure boundary-value problems and periodic solutions. These chapters provide an excellent introduction to the subject. The analysis is always very clear, rigorous and well-organized.

The author takes the reader on a different route via Chapter 2 and 10 through 12. The Cauchy problem, which is ill-posed, is followed by analyticity properties of solutions which are used to study continuous dependence on the data for some ill-posed problems. Chapter 12 contains results of numerical experiments for some of the problems. These chapters are novel. Discussion of such matters is usually brief.

Chapter 13 deals with the inverse problem of measuring time-dependent diffusivity. The measurement problems are formulated by over-specifying the data.

The reader is introduced to moving-boundary problems in Chapters 14, 17 and 18. An elegant analysis of existence/uniqueness and various properties of the free-boundary is given for one-phase Stefan problems.

Finally, Cannon carries out an analysis of various initial-boundary-value problems for the inhomogeneous equation in Chapter 19 (wherein lay the solution of my colleague's problem) using techniques of integral representation and extends it to some quasilinear equations in Chapter 20.

The list of references to the literature covering the period 1800-1982 is truly encyclopaedic (taking up over 120 pages) and well-subclassified.

The author has certainly made all the above topics accessible to the first year graduate student. The mathematical background needed does not extend far beyond the list of results in Chapter 0. A few results on the convergence properties of Foruier series (assumed in Chapter 13) and in real and complex analysis (assumed in Chapter 10) could, profitably, have been included in the list. However, his achievement in leading the reader with such a modest background to such impressive results is remarkable. Considerable care and patience must have been exercised in the preparation of the book.

The book contains a wealth of problems. Some of them are set at the end of a chapter but most appear as proofs left to the reader. They vary in difficulty from straightforward extensions of the text to the fairly challenging. I have only one negative note to sound in this regard - one of the most frequently utilized results in the book (the Extended Comparison Theorem) was relegated to an exercise at the end of Chapter 15. Such an important result should have been part of the text. The last monograph on this topic of which I am aware is that of Widder in 1975 [7]. The scope of that book was a good deal narrower than that of Cannon's and contained no exercises. Cannon's problems provide a welcome pedagogic addition to the literature.

The book is of interest for a variety of reasons. It is classified under the section heading: Analysis. As a textbook on applied analysis it is excellent. Standard theorems are really put to work. The Arzela-Ascoli Tehorem, for example, was used several times in the establishment of an existence proof. There is considerable interest at the present time in the development and application of Boundary-Integral Equation (B.I.E.) methods to dynamic problems in Heat

Transfer [3,4] and related problems. The abstract method which has been analysed in Cannon's book is known in the B.I.E. community as the 'indirect B.I.E. method' and is less popular than the so-called 'direct method' [2]. The material in the book could provide a basis for the numerical analysis of indirect methods and could possibly be extended to cover direct methods also. I am unaware of any work along these lines.

The approach to parabolic equations which has been demonstrated by the author is not, of course, the only one. A different domain of ideas (which, for example, finds application in the Finite Element solution of parabolic equations) is based on a distributional approach (involving concepts of Sobolev spaces and Semigroups). The mathematical apparatus needed to deal with such ideas is a good deal deeper than that required by Cannon [1,5,6]. The quality of production (layout, print, diagrams) is excellent. I counted about thirty errors of various kinds (trivial misprints, references to incorrect or non-existent equations/theorems, a few incorrect statements of a minor nature).

In summary, I would consider this to be among the best books I have read on partial differential equations. Every university library should have a copy.

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"ADVANCED ENGINEERING MATHEMATICS"

By *Ladis D. Kovach*

Published by *Addison-Wesley Publishing Co., Inc.*, 1982, xi + 706 pp. Stg £14.50. ISBN 0-906783-05-4

What is the engineer's role in society? How does mathematics assist the engineer? What mathematical skills does an engineer need? Who is best equipped to teach him these skills? Should he receive a shallow treatment of a great variety of different mathematical topics or a thorough treatment of a few?

These are but a few of the many questions that must constantly occupy the minds of faculty members in any institutions that train future engineers; and anybody intending to write a

mathematics textbook for students of modern engineering science must address himself to them. The resulting book will, inevitably, reflect the author's perceptions of what constitutes a suitable mathematical training for the engineer who will tackle tomorrow's problems.

In the preface of the book under review, the author declares "that *design* is the primary function of an engineer"; and that "a prerequisite for design is *analysis*". He goes on to announce his purpose in writing the book: "our objective is to demonstrate in a number of ways how an engineer might strip a problem of worldly features that are unimportant complexities, approximate the problem by means of a mathematical representation, and analyze *this*." In this respect, the author's intentions are no different to those of writers of similar books in which mathematical modelling is used to come to grips with physical problems.

A number of mathematical techniques that are used in engineering analysis are discussed in the text. The chapter headings may convey some idea of the material covered in the book:

1. First-Order Ordinary Differential Equations;
2. Higher-Order Differential Equations;
3. The Laplace Transformation;
4. Linear Algebra;
5. Vector Calculus;
6. Partial Differential Equations;
7. Fourier Series and Fourier Integrals;
8. Boundary-Value Problems in Rectangular Coordinates;
9. Boundary-Value Problems in Other Coordinate Systems;
10. Complex Variables.