MATHEMATICS EDUCATION

ON TEACHING MATRIX ALGEBRA BY COMPUTER

Bob Critchley and Gordon S. Lessells

Since 1983, an experiment in computer-assisted learning (CAL) has been running at NIHE, Limerick, whereby students on various courses have had the opportunity of learning the basics of Matrix Algebra at a computer terminal.

Origins

In 1983, a project funded by Shannon Free Airport Development Company (SFADCO) was set up, based at NIHE, to create a CAL package called "Costing for Small Business". The aims of the project were (1) to investigate the potential for CAL in management and third-level education and (2) to decide on the feasibility of setting up a company to produce CAL software. A NIHE initiated project, viz. to create a CAL package in Matrix Algebra, was chosen to run in conjunction with "Costing for Small Business".

System

Control Data Corporation (CDC) have been heavily involved with computer-based education using the PLATO system in the United States. The CDC-110 stand-alone microcomputer-based system was chosen for the production of both our courses. In 1983, the CDC-110 system allowed the creation of lessons according to three different models:

(1) Tutorial Learning Model (TLM)
(2) Drill and Practice Model (DPM)
(3) Situation Simulation Model (SSM)

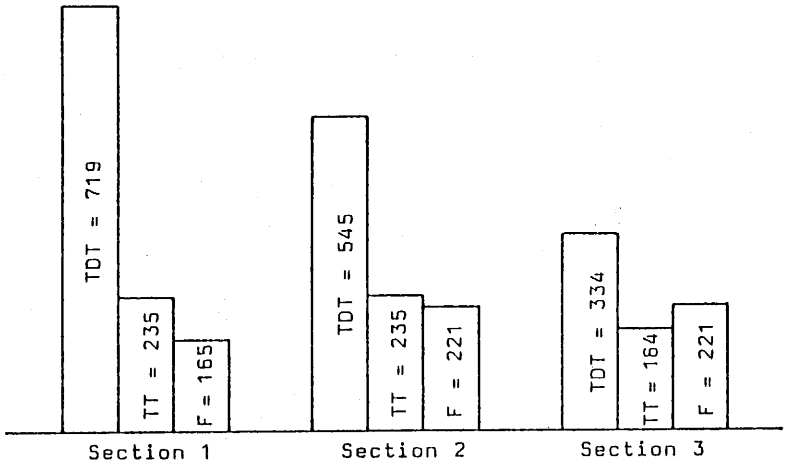
The Tutorial Learning Model was closest to the lecture/tutorial

style of teaching and was chosen for our course. By means of a menu-driven system a lesson could be created as a sequence of frames without using the underlying micro-TUTOR language. However, acquiring a mastery of the command language was almost equivalent to learning a new programming language.

Creation

Two Applied Mathematics students, Donal Crosse and David Nash, on their Co-operative Education placement, were employed by the project for a period of six months to assist us in creating the CAL package. The textual material was written by ourselves, with the design and layout of the frames carried out by Donal and Dave. The material was written in an interactive style in order to retain the attention of students. To this end we were able to take advantage of the touch-sensitive screen. In order not to confront the student with a large quantity of material at once, frames were built up gradually, requiring a key press from the student to proceed. Questions could also be followed by different material according to the response given. By suitable highlighting and animation it was possible to pinpoint the student's attention at the desired section of the screen thus giving the textual material more life than, say, a programmed learning text.

The finished product was a lesson of about six hours duration comprising three sections (1) Matrix Basics, (2) Linear Combinations of Matrices, (3) Multiplication of Matrices. A student is routed through the course having to pass a criterion test at the end of each subsection to allow him/her to proceed. Failure of a criterion test gives the student the option of redoing the previous subsection or reviewing back-up material before taking a new criterion test. Records of time spent and performance in the tests are kept by the computer. In all there were about 600 frames, although each student would only see a fraction of the total. A breakdown of the development time with evidence of the "learning curve" is given below.



TDT - Total Development Time (hours)

TT - Terminal Time (hours)

F - Number of Frames

Reactions to Course

As a first experiment in CAL, matrix algebra was found to be a very suitable topic for a number of reasons:

(1) The topic is widely taught throughout the institute and can be taught independent of applications. The same course can, therefore, be used by both engineering and business students.

(2) The prerequisites were minimal, being a knowledge of Leaving Certificate pass level algebra.

(3) The material lent itself to visual presentation. The use of blackboard and chalk is awkward when large matrices are involved.

(4) The material involved understanding concepts as well as

acquiring calculating skills.

Reaction of students to the course was gauged by means of interviews and questionnaires. Some of the observed benefits were:

(1) Students could go at their own pace.

(2) 12-hour access to the computers allowed students to choose their own study times.

(3) Some students also felt that the interactive nature of the course allowed them to learn better than by traditional methods.

On the debit side, some students were more concerned about the absence of a tutor whereas other students were more concerned about repetition of the same material because of failure in a criterion test. Various tests and subsequent exam results have indicated that the course provided a good foundation for further work on Matrix Algebra. We do, however, believe that the CAL approach is only suitable for a limited number of areas of mathematics, vectors, more matrices, probability, graph theory being four suggestions for possible investigation.

CDC - DEC

The original CDC system had a number of disadvantages:

(1) The lesson models were too restrictive.

(2) The use of floppy disks was unsuitable for use by large numbers.

(3) The system was slow and noisy.

(4) Customer service was poor.

(5) The system was incompatible with other micros.

For these and other reasons, the institute has adopted the Digital Equipment Corporation's DEC PRODUCER system for its

CAL laboratory. The original course has been reprogrammed and is now available in colour to students on DEC PRO-350 microcomputers or DEC VT 220 terminals linked to a Microvax 11. The increased computing power has reduced the time taken by students and the greater availability has meant that large classes can now be accommodated. Anyone interested in seeing (or purchasing!) the course should contact the authors.

Finally, we wish to acknowledge the assistance of Dr Joe Smyth, Dr Mark Burke, Eamonn Murphy, Mary Davern, Anna Kinsella and Brenda Sugrue who have all made valuable contributions along the way.

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UNDERGRADUATE PROJECTS IN GROUP THEORY: AUTOMORPHISM GROUPS

T. Porter

In my earlier note [3], I described a project undertaken by a 3rd year student involving commutativity ratios. The basic tools needed were an intuitive idea of a presentation of a group, and some modular arithmetic. That project was the equivalent of a half paper in the final exams. The following year the system was modified and projects were enlarged so as to be equivalent to a full paper in the final exams. Here I will describe briefly a project involving calculation of automorphism groups from a presentation. The groups studied were the dihedral groups, which have a fairly easy presentation readily available:

$$D_n = \langle x,y : x^n = y^2 = (xy)^2 = e\rangle \text{ for } n \geq 3.$$

The idea of the project was as follows:

If $\alpha: D_n \to D_n$ is an automorphism then

$$\alpha(x) = x^i y^j$$
$$\alpha(y) = x^k y^\ell$$

for some $0 \leq i,j \leq n-1$ and $0 \leq j,\ell \leq 1$ since any element of D_n has a representation in the form $x^a y^b$, with $0 \leq a < n$, $0 \leq b < 2$. If one wants to build automorphisms, therefore, one may attempt to do so by picking "suitable" i,j,k,ℓ. Of course saying that α is a homomorphism and specifying $\alpha(x)$ and $\alpha(y)$ will say where each $x^a y^b$ is to go provided that the relations are compatible with the choice of i,j,k,ℓ. By this we mean that

$$x^n = e \Rightarrow (\alpha(x))^n = e \text{ that is } (x^i y^j)^n = e$$

$$y^2 = e \Rightarrow (\alpha(y))^2 = e \text{ that is } (x^k y^\ell)^2 = e$$