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# ASPECTS OF HIPPARCOS

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## 1. INTRODUCTION

Some aspects of the HIPPARCOS space astrometry mission are presented in this article. The objectives of the mission and its broad principles of operation are described in Section 2. In Section 3, system level analyses of the mission, to which our company has contributed over the past few years, are outlined. Finally, in Section 4, as an illustration of the work, a specific, relatively simple, problem is discussed.

## 3. HIPPARCOS

HIPPARCOS is a space astrometry mission, sponsored by the European Space Agency (ESA), which is scheduled for launch in 1988. Its objective is to measure the astrometric parameters (positions, proper motions, trigonometric parallaxes) of about 100,000 pre-selected stars to a (very high) accuracy of 0.002 arcseconds.

The basic principle of measurement is to scan, continuously and systematically, the entire sky with a telescope capable of accurately measuring the angles between stars separated by a large angle. It is possible, by numerically combining several millions of such angular measurements, to derive the required astrometric parameters. The period of data collection is to be $2\frac{1}{2}$ years.

The telescope is equipped with two fields of view (FOVs) to enable measurement of the angles between widely separated stars. Each FOV is of dimension 0.9° x 0.9°. The angle between the FOVs, called the *basic angle*, is denoted by $\gamma = 58^r$

The FOVs scan the entire celestial sphere through a combination of two motions (see Fig. 1):

(a) A short period spin about the $Z_G$-axis (rate R = 11.25 rotations/day).

(b) A long period revolution (precession of the $Z_G$ axis) which describes an axisymmetric cone about the line joining the satellite and sun. The half-cone angle is called the revolving scanning angle and is denoted by $\zeta$ (= 43°). The average precession rate is K (=6.4 revolutions/year).

There is a modulating grid at the focal plane of the telescope which, together with a photon counting detector, encodes the movement of a star as it crosses a FOV. This constitutes the primary instrument. In addition, there is another detector (called a 'star mapper') placed in the focal plane. Its purpose is to provide data for control of the satellite's attitude and to fulfil a supplementary scientific mission (named TYCHO). (By attitude is meant the pointing directions of the $Z_G$ and X axes - see Fig. 1).

## 3. GENERAL SYSTEM ANALYSIS

### Selection of Key Parameters

The values chosen for parameters $\zeta$, K and R are limited by certain technical considerations. For example, the electrical power supply (solar panels) depends on $\zeta$ while a low value of K makes for ease of manoeuvrability; the choice of R is limited by data rates and on-board computer capability.

There are scientific constraints, also. These include a requirement for uniform sky coverage, optimisation of global accuracy and minimisation of interruptions (occultations) due to earth and moon. There are similar considerations for the



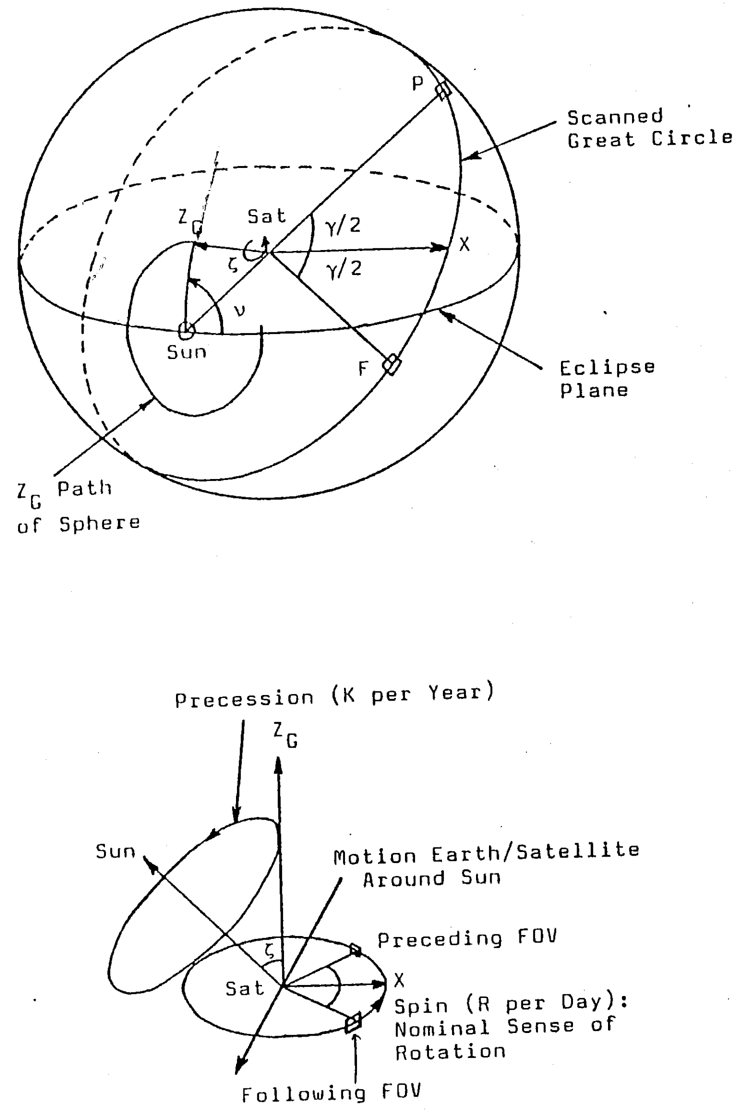


FIGURE 1: HIPPARCOS SCANNING MOTIONS

choice of $\gamma$; for example, integral divisors of $\pi$ are found to exhibit undesirable behaviour in the data reduction process.

## Accuracy Analysis

Accuracy analysis makes up a major part of systems activity. Its purpose is to assess the impact of different error sources on overall accuracy. Such assessments enable design trade-offs to be made, for example. Some of the major error sources considered are photon statistical noise, background noise, high frequency attitude jitter and irregularities of the grid.

## Photometric Calibration

While we have contributed to the two foregoing topics, mainly at a computational level, our main activity has concerned *in-orbit calibration* of the satellite's payload. (Prelaunch calibrations are quite distinct.) There are two principal topics, photometric calibration and geometric calibration.

As an illustration of photometric calibration, consider

$$I_o = CI_{pf} \qquad (1)$$

where $I_o$ represents the photoelectron count rate observed by an instrument and $I_{pf}$ the incident photon flux. The objective of the calibration is to estimate C, the instrument sensitivity.

In practice, C is not a simple constant. It may depend, for example, on position in the field of view $(\eta, \xi)$, on star colour (B - V), on time (t) and on count rate (non-linear effect). Thus a simple form for C might be

$$C = C_0 + C_1\eta + C_2\xi + C_3(B - V)$$

$$+ C_4\eta(B - V) + C_5\xi(B - V) + C_6 t + C_7 I_o \qquad (2)$$



Therefore, the calibration task is to estimate parameters $C_0$ to $C_7$. A weighted least squares procedure is the method used for this estimation.

A major part of the work is to assess the performance of the calibration method. The main results of such an assessment are the accuracy achievable for a measurement period of given duration (i.e. for a given volume of data) and the appropriate form for function C. The assessment takes account of measurement error models, of predicted instrument response and of *a priori* errors on star magnitude and colour.

## Geometric Calibration

Each star in a field of view is assigned a longitudinal ($\eta$) and transverse ($\xi$) coordinate; these define the *field* (sky) position of the star. This is illustrated in Fig. 2, which distinguishes preceding (p) and following (f) fields. For each star, there is a corresponding star image on the detector grid. This image is assigned *grid* coordinates (G,H).

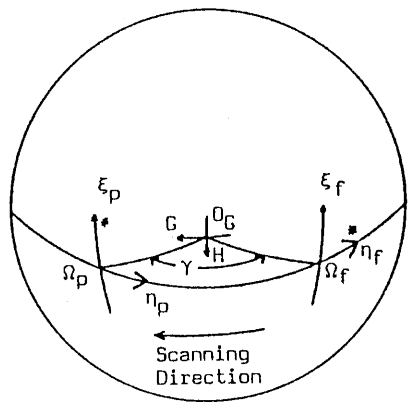


FIGURE 2: FIELD AND GRID COORDINATES

There is a mapping between field and grid which can be described in polynomial form. Thus, for the longitudinal grid coordinate one has

$$G = \sum_{n=0} \sum_{m=0} a_{mn}^{(\alpha)} \eta^m \xi^n \qquad (3)$$

where $\alpha$ = p or f according to field of view.

The various terms of (3) may be associated with such effects as grid defocusing and in-plane displacements, grid rotation and tilts and telescope mirror deformations. The basic angle ($\gamma$) can be included in (3), in the terms $a_{00}^{(p)}$, $a_{00}^{(f)}$.

The main part of the mapping, called the nominal field to grid transformation, is known pre-launch. However, it is an in-orbit calibration task to estimate the additional distortion induced post-launch.

The above polynomial form describes *large scale* distortions. In addition, it is necessary to determine *medium scale* distortions. The latter are described by a large matrix of components ( ≈ 150 x 150). However, the calibration method devised takes account of the good pre-launch knowledge of these components and, thereby, reduces the measurement time which would otherwise be necessary.

Among other geometric calibration tasks may be noted that of chromaticity calibration. In the present context, chromaticity refers to the displacement of a star image with respect to the image position of a star of average colour.

## 4. A SPECIFIC PROBLEM

### Method

The main in-orbit calibration activities are carried out during the commissioning period. This commences some days



after launch and lasts about one month. However, prior to this, there is an initialisation phase during which attitude control of the satellite is acquired. As part of this process, a first calibration of the basic angle ($\gamma$) and of grid rotation ($\theta$) is required; this topic is discussed in what follows.

The star mapper is the only detector operational during the initialisation phase. Therefore, its measurements must form the basis for the calibration method. Essentially, this detector measures the time at which a star crosses a particular reference line in the field of view (preceding or following, as appropriate). The distance between the reference lines is exactly $\gamma$ (see Figs 2 and 3).

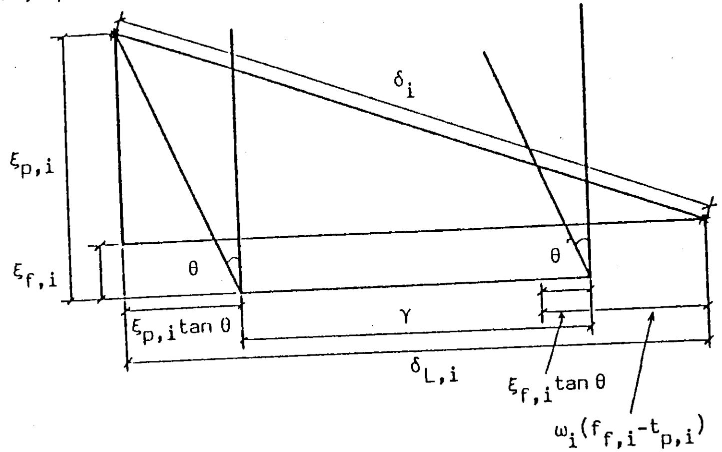


FIGURE 3: STAR SEPARATION

The method is based on measurements on a set of star pairs. Each pair is such that one member crosses the preceding reference line at approximately the same time as the other member crosses the following reference line (see Fig. 2). Hence, the separation between the pair is, approximately, $\gamma$.

Consider the $i^{th}$ such pair, as in Fig. 3. Let the *a priori* value for their separation be $\delta_i$. Thus, its longitudinal (along scan) component is

$$\delta_{L,i} = (\delta_i^2 - (\xi_{p,i} - \xi_{f,i})^2)^{\frac{1}{2}} \qquad (4)$$

where $\xi_{p,i}$, $\xi_{f,i}$ are transverse coordinates.

Let $t_{p,i}$, $t_{f,i}$ be the transit times for the preceding and following stars, respectively. Then, if $\omega_i$ is the rotation rate, one has

$$\delta_{L,i} = \gamma + \omega_i(t_{f,i} - t_{p,i}) + \tan\theta\,(\xi_{p,i} - \xi_{f,i})$$

In fact, θ is small so that one may write

$$\gamma + \theta(\xi_{p,i} - \xi_{f,i}) = \delta_{L,i} - \omega_i(t_{f,i} - t_{p,i}) + \varepsilon_i \qquad (5)$$

The added term, $\varepsilon_i$, represents error (from a number of sources).

The measurements from a number, N say, of such star pairs are collected, each giving rise to an equation of form (5). A weighted least squares method is applied to this set of equations to obtain estimates $\hat{\gamma}$ and $\hat{\theta}$.

## Assessment

In order to assess the method, a number of simplifying, but not unrealistic, assumptions may be made. Thus, one may assume that observations are uncorrelated and that

$$\mathrm{Var}\ (\varepsilon_i) = \sigma^2 \quad \forall\, i \qquad (6)$$

Further, one may assume that $\xi_{p,i}$, $\xi_{f,i}$ are both random variables uniformly distributed in (-a,a) (a = 20 arcminutes for star mapper). It then follows, approximately, that

$$\hat{\gamma} = \frac{1}{N}\sum y_i \qquad (7)$$

$$\hat{\theta} = \frac{1}{N}\left(\frac{3}{2a^2}\right)\sum y_i(\xi_{p,i} - \xi_{f,i}) \qquad (8)$$

where

$$y_i = \delta_{L,i} - \omega_i(t_{f,i} - t_{p,i}) \qquad (9)$$



Moreover,

$$\mathrm{Var}\ (\hat{\gamma}) = \sigma^2/N \qquad (10)$$

$$\mathrm{Var}\ (\hat{\theta}) = \left(\frac{3}{2a^2}\right)\sigma^2/N \qquad (11)$$

and

$$\mathrm{Cov}\ (\hat{\gamma},\hat{\theta}) = 0 \qquad (12)$$

Thus, the achievable accuracy depends (unsurprisingly) on the ratio

$$\mu = \sigma^2/N \qquad (13)$$

An approximate error analysis of the right-hand side of (5) yields

$$\sigma^2 = a + bT_s^2 \qquad (14)$$

in which estimated values are available for a and b. $T_s$ is the average interval between transits of a *suitable* star pair,

$$T_s = \overline{t_{f,i} - t_{p,i}} \qquad (15)$$

Let the total number of candidate calibration stars, assumed uniformly distributed in the sky, be M. Then, the density per square degree is

$$\rho = M\pi/4(180^2) \qquad (16)$$

Noting that the mean spin rate is 168.75° per hour and that the star mapper width is 40 arcminutes, it follows that an area

$$A = (168.75/3600)(40/60) = 0{,}03125 \qquad (17)$$

square degrees is swept out in 1 second. Hence, the average rate of arrival of a candidate star in the star mapper is given by

$$\lambda = A\rho = 0.03125\rho \qquad (18)$$

A minimum separation time of 10 seconds, between members of a star pair, is necessary to avoid ambiguity in identification. On imposing this constraint and on letting the maximum separation time be $t_{max}$, it can be shown that

$$T_p = \frac{1}{\lambda}[e^{-60\lambda} - e^{-2\lambda(t_{max}+20)}]^{-1} \qquad (19)$$

where $T_p$ is the average interval between *suitable* star pairs. Moreover, one may show that

$$T_s = \frac{1}{\lambda} + [10e^{-10\lambda} - t_{max}e^{-\lambda t_{max}}][e^{-10\lambda} - e^{-\lambda t_{max}}]^{-1} \qquad (20)$$

The total number of *suitable* star pairs in a given period $T_{tot}$ may then be calculated as

$$N = T_{tot}/(T_p + T_s) \qquad (21)$$

It is clear from the foregoing that the two elements of μ (Equation (13)) depend on $t_{max}$. In order to optimise the method's performance one seeks to minimise μ. However, it is clear from the nature of the dependencies on $T_s$ that there are conflicting objectives (of minimising $\sigma^2$ and maximising N). The value of $t_{max}$ (and, hence, of $T_s$) which gives the best compromise between these objectives defines a *suitable* star pair.

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ERRATUM

In my paper "Capacities, Analytic and Other", *IMS Newsletter*, 13, pp. 48-56, the symbol $|| \, |\nabla\phi| \, ||_{L_1}$ appearing in line 6 of p. 54 should be replaced by $|| \, |\nabla\phi| \, ||_{H_1}$. Then in lines 10-13 of that page $W^{1,1}$ should be redefined as the space of $L_1$ functions with $H_1$ distributional derivatives.

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