Course 3413 — Group Representations
Sample Paper III

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2 hour paper

Attempt 3 questions. (If you attempt more, only the best 3 will be counted.) All questions carry the same number of marks. Unless otherwise stated, all groups are compact (or finite), and all representations are of finite degree over $\mathbb{C}$.

1. Define a *group representation*.

   What is meant by saying that a representation α is *simple*? Determine all simple representations of S_3. from first principles.

   Determine the characters of S_4 induced by the simple characters of S_3. Hence or otherwise draw up the character table for S_4

2. Show that the number of simple representations of a finite group G is equal to the number s of conjugacy classes in G.

   Determine the conjugacy classes in A_4 (formed by the even permutations in S_4), and draw up its character table.

   Determine also the representation-ring for this group, ie express the product $\alpha\beta$ of each pair of simple representation as a sum of simple representations.

More questions overleaf!

3. Determine all groups of order 8 (up to isomorphism); and for each such group G determine as many simple representations (or characters) of G as you can.

4. Determine the conjugacy classes in SU(2); and prove that this group has just one simple representation of each dimension.

   Define a covering homomorphism

   $$\Theta : \mathrm{SU}(2) \to \mathrm{SO}(3);$$

   and hence or otherwise show that SO(3) has one simple representation of each odd dimension $1, 3, 5, \ldots$.