

## Course 424

## Group Representations II

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Arts Block A2039 Friday, 20 January 1989 15.45–17.45

Answer as many questions as you can; all carry the same number of marks.

Unless otherwise stated, all representations are finite-dimensional over  $\mathbb{C}$ .

1. Explain carefully and fully what is meant by the statement that there exists a unique left-invariant measure  $\mu$  on every compact group G; and sketch the proof of this statement.

Determine the measure in  $\mathbf{SU}(2)$  of the set of all matrices having eigenvalues  $e^{\pm i\theta}$ , where  $0 \le \theta \le \pi/4$ .

2. Prove that every simple representation of a compact abelian group A is 1-dimensional.

Determine the simple representations of U(1).

3. Determine the conjugacy classes in  $\mathbf{SU}(2)$ .

Prove that SU(2) has just one simple representation of each dimension  $1, 2, \ldots$ ; and determine the character of this representation.

4. Determine the conjugacy classes in SO(3).

Show that there exists a covering  $\Theta$  :  $\mathbf{SU}(2) \to \mathbf{SO}(3)$ , ie a surjective homomorphism with finite kernel.

Hence or otherwise determine the simple characters of SO(3).

Show that all the representations of SO(3) are real.