Course 424

Group Representations II

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Exam Hall   Tuesday, 4 April 1997   15:15–16:45

Answer as many questions as you can; all carry the same number of marks.
All representations are finite-dimensional over $\mathbb{C}$.

1. What is meant by an *invariant measure* on a compact group G? Sketch the proof that every compact group G carries such a measure. To what extent is this measure unique?

2. Prove that every simple representation of a compact abelian group is 1-dimensional and unitary.

   Determine the simple representations of $\mathbf{U}(1)$.

3. Determine the conjugacy classes in $\mathbf{SU}(2)$. Prove that $\mathbf{SU}(2)$ has just one simple representation of each dimension $m = 1, 2, 3, \ldots$; and determine the character of this representation.

4. Show that there exists a surjective homomorphism

   $$\Theta : \mathbf{SU}(2) \to \mathbf{SO}(3)$$

   with finite kernel.

   Hence or otherwise determine all simple representations of $\mathbf{SO}(3)$.