Course 424

Group Representations II

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Arts Block A2039 Friday, 20 January 1989 15.45–17.45

Answer as many questions as you can; all carry the same number of marks.
Unless otherwise stated, all representations are finite-dimensional over $\mathbb{C}$.

1. Explain carefully and fully what is meant by the statement that there exists a unique left-invariant measure μ on every compact group G; and sketch the proof of this statement.

   Determine the measure in $\mathbf{SU}(2)$ of the set of all matrices having eigenvalues $e^{\pm i\theta}$, where $0 \le \theta \le \pi/4$.

2. Prove that every simple representation of a compact *abelian* group A is 1-dimensional.

   Determine the simple representations of $\mathbf{U}(1)$.

3. Determine the conjugacy classes in $\mathbf{SU}(2)$.

   Prove that $\mathbf{SU}(2)$ has just one simple representation of each dimension $1, 2, \ldots$; and determine the character of this representation.

4. Determine the conjugacy classes in $\mathbf{SO}(3)$.

   Show that there exists a covering $\Theta : \mathbf{SU}(2) \to \mathbf{SO}(3)$, ie a surjective homomorphism with finite kernel.

   Hence or otherwise determine the simple characters of $\mathbf{SO}(3)$.

   Show that all the representations of $\mathbf{SO}(3)$ are real.