UNIVERSITY OF DUBLIN

XMA34691

TRINITY COLLEGE

Faculty of Engineering, Mathematics
and Science

school of mathematics

SS Theoretical Physics **Trinity Term 2010**
JS & SS Mathematics

Module MA3469 Practical Numerical Simulations — SAMPLE PAPER

DAY, DATE PLACE TIME: 2 HOURS

Dr. D. Grigoriev

Attempt THREE questions.

All questions carry equal marks.

Log tables are available from the invigilators, if required.

Non-programmable calculators are permitted for this examination,—please indicate the make and model of your calculator on each answer book used.

1. Numerical evaluation of the integral

$$I = \int_a^b f(x)\,dx$$

is equivalent to numerical integration of the following first-order ordinary differential equation:

$$y' = f(x). \tag{1}$$

Formulate the proper initial value problem for differential equation (1) which will give the value of I. Write down the appropriate finite difference schemes which solves the differential equation (1) with the fixed-step Euler, 2^{nd}-order Runge-Kutta and 4^{th}-order Runge-Kutta method *(the actual exam paper shall have RK4 formulas here)*. For all three schemes, find in general form both the numerical solution of the differential equation (1) and the value of I. For the first two schemes, determine the order of accuracy of the results.

2. Formulate the proper initial/boundary-value problem for the following parabolic PDE in 2 spatial dimensions:

$$\frac{\partial f}{\partial t} = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

on a symmetric spatial grid. Derive Crank-Nicolson finite-difference scheme for the equation, estimate the accuracy of the scheme and obtain its stability condition.

3. Let random numbers ξ be uniformly distributed on the interval $0 < \xi \le 1$.

   (a) Find the function $G(\xi)$ such that the random numbers $\xi_G = G(\xi)$ have a given distribution $P(\xi_G)$. Calculate $G(\xi)$ for $P(\xi_G) = 1 - |\xi_G|$, $|\xi_G| \le 1$.

   (b) Derive the Box-Muller algorithm for obtaining a pair of normally distributed random numbers ξ_n from a pair of uniform random numbers ξ.

4. Consider N-point Richardson polynomial extrapolation of a set of numerical solutions $\bar{y}(t+H, h_i)$, obtained at the point $t+H$ by a symmetric 2^{nd}-order method with the time step $h_i = H/n_i$, where $i = 1 \ldots N$ and n_i are positive integer numbers:

$$\bar{y}(t+H, h_i) = y(t+H) + \sum_{k=1}^{N} e_k(t+H) h_i^{2k} + \mathcal{O}(H^{2N+2})$$

($y(t+H)$ is the exact solution). Prove that the truncation error for the extrapolation result $\bar{y}_N(t+H)$ can be written as

$$\epsilon_N(t+H) = y(t+H) - \bar{y}_N(t+H) = \frac{(-1)^N}{n_1^2 \ldots n_N^2} e_N(t+H) H^{2N} + \mathcal{O}(H^{2N+2}).$$