Problem Solving Set 19

$21 \ \mathrm{July} \ 2012$

1. Let 0 < a < b. Prove that

$$\int_{a}^{b} (x^{2} + 1)e^{-x^{2}} dx \ge e^{-a^{2}} - e^{-b^{2}}.$$

2. If $n \in \mathbb{N}$ has the property that for all $x \in \mathbb{Z}$ there exists $y \in \mathbb{Z}$ such that $x^2 + y^2 \equiv 1 \pmod{n}$, show that n divides 12.