Problem Solving
Set 8

11 July 2012

1. Let C be a nonempty closed bounded subset of the real line and $f : C \to C$ be a non-decreasing continuous function. Show that there exists a point $p \in C$ such that $f(p) = p$.

   (A set is closed if its complement is a union of open intervals. A function g is non-decreasing if $g(x) \leq g(y)$ for all $x \leq y$.)

2. *Modified* Let $P(x)$ be a polynomial with integer coefficients of the form

   $$P(x) = x^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0.$$

   Show that if the degree n is odd, the constant term a_0 is odd, and $P(x)$ has an odd number of odd coefficients, then $P(x)$ has at least one irrational real root.