

# Problem Solving

## Set 6

09 July 2012

1. Let  $f$  be a polynomial of degree 2 with integer coefficients. Suppose that  $f(k)$  is divisible by 5 for every integer  $k$ . Prove that all coefficients of  $f$  are divisible by 5.
2. (a) Given a triangle  $ABC$  show that there is a unique triangle  $PQR$  with the points  $P, Q, R$  on the sides  $BC, CA, AB$ , and the edges  $QR, RP, PQ$  parallel to  $BC, CA, AB$ , respectively.  
(b) Suppose we inscribe a third triangle in the same way inside  $PQR$ , and a fourth triangle inside this one, and so on. Show that the areas of the triangles form a geometric sequence.