

Problem Solving

Set 5

08 July 2012

1. Let f be a real-valued function with $n + 1$ derivatives at each point of \mathbb{R} . Show that for each pair of real numbers $a, b, a < b$, such that

$$\ln \left(\frac{f(b) + f'(b) + \cdots + f^{(n)}(b)}{f(a) + f'(a) + \cdots + f^{(n)}(a)} \right) = b - a$$

there is a number c in the open interval (a, b) for which

$$f^{(n+1)}(c) = f(c).$$

2. Given one million non-zero digits $a_1, a_2, \dots, a_{1000000}$ (ie each $a_i \in \{1, 2, \dots, 9\}$) show that at most 100 of the million numbers

$$a_1, a_1 a_2, a_1 a_2 a_3, \dots, a_1 a_2 a_3 \dots a_{1000000}$$

are perfect squares