Problem Solving Set 3

06 July 2012

- 1. Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ be given by $f(x, y) = (x^2 y^2)e^{-x^2 y^2}$
 - (a) Prove that f attains its minimum and maximum.
 - (b) Determine all points (x, y) such that

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = 0$$

and determine for which of them f has global or local minimum or maximum.

2. A sequence u(n) satisfies the relation

$$u(n+2) = [u(n+1)]^2 - u(n),$$

with u(1) = 13 and u(2) = 45. Prove that 2012 divides infinitely many terms of the sequence.