Problem Solving Set 2

05 July 2012

- 1. (a) Let A be a $n \times n$, $n \ge 2$, symmetric, invertible matrix with real positive elements. Show that $z_n \le n^2 - 2n$, where z_n is the number of zero elements in A^{-1} .
 - (b) How many zero elements are there in the inverse of the $n \times n$ matrix

| 1 | ′ 1 | 1 | 1 | 1 | 1 | |
|---|------------|---|---|---|-------|---|
| | 1 | 2 | 2 | 2 | 2 | |
| | 1 | 2 | 1 | 1 | 1 | 2 |
| | 1 | 2 | 1 | 2 | 2 | : |
| | | | | | | |
| | 1 | 2 | 1 | 2 |) | |

2. Let $a_1, a_2, ..., a_n$ be distinct integers. Show that

$$f(x) = (x - a_1)(x - a_2)...(x - a_n) - 1$$

is irreducible over the rationals.