

Problem Solving

Set 1

04 July 2012

1. Let $f \in C^1(a, b)$, $\lim_{x \rightarrow a+} f(x) = +\infty$, $\lim_{x \rightarrow b-} f(x) = -\infty$ and $f'(x) + f^2(x) \geq -1$ for $x \in (a, b)$. Prove that $b - a \geq \pi$ and give an example where $b - a = \pi$.
2. A collection of subsets of $\{1, 2, \dots, n\}$ has the property that each pair of subsets has at least one element in common. Prove that there are at most 2^{n-1} subsets in the collection.