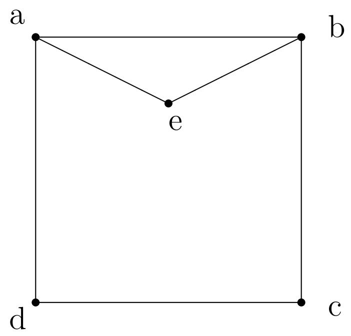
Irish Intervarsity Mathematics Competition 1993

University College Galway

*A*nswer all ten questions

1. Evaluate $\sum_{j=2}^{\infty}\left(\sum_{i=2}^{\infty}\frac{1}{i^j}\right)$.

2. $abcd$ is a square and e is a point inside the square such that $|ae| = |be|$ and $e\hat{b}a = 15^\circ$. Show that dec is an equilateral triangle.



3. If x, y and n are all natural numbers and $t = x^{4n} + y^{4n} + x^{2n}y^{2n}$ is a prime number, find all possible values of t.

4. Construct, with proof, a non–constant arithmetic progression of positive integers which contains no squares, cubes or higher powers of integers.

5. If ${}^nC_r$ is the number of combinations of n objects r at a time, find the value of $\sum_{r=0}^{n}\left(\frac{1}{r+1}\right){}^nC_r$.

6. If x and y are real numbers, find all solutions of the equation

$$(x^4+1)=(x^2)(2^{1-y^2}).$$

7. Find a formula for the nth derivative of the function $f(x)=\dfrac{q}{x^2+x^2}$ where a is a constant.

8. If abc is an obtuse angled triangle with $b\hat{a}c>\frac{\pi}{2}$ prove that

$$54|ac|^3|ab|^3|\cos b\hat{a}c|\leq|bc|^6$$

and find conditions under which equality holds.

9. Show that there do *not* exist polynomials $f(x)$ and $g(x)$ such that

$$e^x=\frac{f(x)}{g(x)}\quad\text{for all }x.$$

10. Find the real factors of $x^4+y^4+(x+y)^4$.