Exercise 6

Polynomial Rings

In exercises 1–10 factorize the given polynomial in $\mathbb{F}_2[x]$.

* 1. $x^2 + x$

* 2. $x^2 + 1$

* 3. $x^2 + x + 1$

** 4. $x^3 + x^{2\text{‘}} + x + 1$

** 5. $x^4 + x^3 + x^{2\text{‘}} + x + 1$

** 6. $x^5 + 1$

** 7. $x^5 + x^3 + 1$

** 8. $x^8 + 1$

*** 9. $x^8 + x + 1$

*** 10. $x^9 + 1$

** 11. Determine the irreducible polynomials of degrees 1,2 and 3 over $\mathbb{F}_2$.

*** 12. Determine the irreducible polynomials of degree 4 over $\mathbb{F}_2$.

*** 13. How many irreducible polynomials are there of degree 5 over $\mathbb{F}_2$?

** 14. Determine the irreducible polynomials of degree 2 over $\mathbb{F}_3$.

*** 15. Determine the irreducible polynomials of degree 3 over $\mathbb{F}_3$.

*** 16. How many irreducible polynomials are there of degree 4 over $\mathbb{F}_3$?

** 17. Determine the irreducible polynomials of degree 2 over $\mathbb{F}_5$.

** 18. Determine the irreducible polynomials of degree 2 over $\mathbb{F}_7$.

** 19. Show that an irreducible polynomial over $\mathbb{R}$ is of degree 1 or 2.

** 20. Determine the irreducible polynomials over $\mathbb{C}$.

In exercises 21–25 determine if the given polynomial is irreducible over $\mathbb{Q}$.

** 21. $x^2 + x + 1$

** 22. $x^3 + 2x + 1$

*** 23. $x^4 + 1$

*** 24. $x^4 + 2$

*** 25. $x^4 + 4x^3 + 1$

**** 26. Can you find polynomials $f(x), g(x), h(x) \in \mathbb{F}_2[x]$ such that $f(x)^3 + g(x)^3 + h(x)^3 = 0$?

In the remaining exercises $k[[x]]$ denotes the ring of formal power series

$$a_0 + a_1 x + a_2 x^2 + \cdots \quad (a_i \in k)\,?$$

** 27. Determine $\frac{1}{1-t+t^2}$ in $\mathbb{F}_1[[x]]$.

** 28. Show that $k[[x]]$ is an integral domain.

** 29. Determine the invertible elements in $k[[x]]$.

*** 30. Does unique factorisation hold in the ring $k[[x]]$?