Exercise 2

In exercises 1–10 determine whether the given sum over $\mathbb{N}$ is convergent or not:

* 1. $\sum_n \frac{1}{n^{1/2}}$

* 2. $\sum_n \frac{1}{n^{3/2}}$

** 3. $\sum_n \frac{1}{n \ln n}$

** 4. $\sum_n \frac{1}{n \ln^2 n}$

** 5. $\sum_n \frac{\ln n}{n^2}$

* 6. $\sum_n \frac{(-1)^n}{n}$

** 7. $\sum_n \frac{(-1)^n}{n^{1/2}}$

** 8. $\sum_n \frac{\cos n}{n}$

*** 9. $\sum_n \frac{\tan n}{n}$

** 10. $\sum_n \sin n$

In exercises 11–13 determine whether the given sum over the primes is convergent or not:

** 11. $\sum_p \frac{1}{p \ln p}$

*** 12. $\sum_p \frac{(-1)^p}{p}$

*** 13. $\sum_p \frac{(-1)^p}{\sqrt{p}}$

**** 14. Show that there are an infinity of primes of the form $3m+2$, where m is a positive integer.

**** 15. Show that there are an infinity of primes of the form $6m-1$.

***** 16. Show that if n is even then any prime factor of n^2+1 is of the form $4m+1$. [This may require knowledge of quadratic residues, which will be met in Chapter 9.]
Hence show that there are an infinity of primes of the form $4m+1$.

*** 17. Determine $\zeta(2)$.

**** 18. Determine $\zeta(4)$.

**** 19. Show that the two forms of the Prime Number Theorem, $\pi(x) \sim \frac{x}{\log x}$ and $p_n \sim n \log$ are equivalent, ie each can be derived from the other.

*** 20. Factorise your 8-digit college ID. [The Linux machines on the maths system, eg hamilton, have a program "factor" (or possibly "/usr/games/factor").]

** 21. Estimate $\sum_{n=10}^{100} \frac{1}{n^2}$.

*** 22. Estimate the probability that there is no prime among the college ID's of a class of 60.