## Number Theory — Course MA2317

## Timothy Murphy

First Semester 2016–2017

Number theory is often called "The Queen of Mathematics". Number theory can be divided, very roughly, into 3 parts:

- Elementary number theory
- Algebraic number theory
- Analytic number theory

This course is concerned, for the main part, with elementary number theory, although we shall dip into the simplest topic in algebraic number theory, namely quadratic number fields. We shall also mention, without proof, the two basic results of analytic number theory, namely the Prime Number Theorem and Dirichlet's Theorem on primes in arithmetic sequences.

We begin with the Fundamental Theorem of Arithmetic, Euclid's Theorem that every natural number n > 0 is uniquely expressible as a product of primes. (This result is so familiar that one can easily overlook the subtlety of the proof, and the enormous step taken by Euclid or his school in establishing it.)

Elementary number theory is, to a large extent, the study of prime numbers. As a kind of game to go with the course, we shall join the hunt for the next largest prime number. This is certain to be a Mersenne prime, due to the Lucas-Lehmer test which can tell us whether enormously large Mersenne numbers  $2^p - 1$  (where p is a prime) are prime or not. We shall be able to establish, as an exercise in quadratic number fields, the validity of this test.

On-line notes for the course are available in http://www.maths.tcd.ie/ pub/Maths/Courseware/NumberTheory/.

The course will be examined at the end of the year. Continual Assessment will account for 20% of the marks for the course. A large number of exercises will be found in the course notes. Each exercise is given 1–5 stars, according to difficulty. Full marks for Continual Assessment will be attained if problems totalling 200 stars are completed.

## Addendum

**Exercises** Keep a copy of your answers to the exercises you complete. I shall ask to see a small sample of them at the end of the year.

Don't send me the answers during the year. But do send me a list of the exercises you have completed from time to time. *Please follow the format below exactly with these lists:* Send them to tim@maths.tcd.ie with Subject MA2317 and with contents in the following form:

Exercise 03 1,3,5 Exercise 07 1,2,3,4,5,6,10

. . .

If you have already sent a list you don't need to duplicate the exercises already listed.

Name and email address Please send email to tim@maths.tcd.ie with Subject MA2317, and the following information, again following the exact format below

> Name, eg Sean Smith College ID (8-digit number) Email address (you need not add @tcd.ie) Mobile number

Your mobile number is not required, and will only be used if the need arises. My number is: 086 233 6090.

Mailing list I am setting up a mailing list for the course. I will use it to send you information on the course. Anyone registered for the course can also send email to those on the list, by addressing it to http://www.maths.tcd.ie/mailman/NumberTheory.

This should be reserved for topics strictly relevant to the course, and only email of a reasonably small size will be accepted.

Course notes Lecture notes and other related information can be found at www.maths.tcd.ie/pub/Maths/Courseware/NumberTheory/ and in the folder NumberTheory/2016.