

Course 424

Group Representations IIIa

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Arts Block A2039 Friday, 3 February 1989 15.00–17.00

Answer as many questions as you can; all carry the same number of marks.

Unless otherwise stated, all Lie algebras are over \mathbb{R} , and all representations are finite-dimensional over \mathbb{C} .

1. Define a *Lie algebra* L , and a *representation* of L .
Explain how a representation α of a linear group G gives rise to a representation of its Lie algebra LG .
Show that if 2 Lie algebras have the same complexification then their representations correspond. Hence or otherwise show that all the representations of $\mathfrak{sl}(2, \mathbb{R})$ are semisimple.
2. Define the *adjoint representation* \mathbf{ad} of a Lie algebra L (verifying that it is indeed a representation).
Define also the adjoint representation \mathbf{Ad} of the linear group G in LG . What is the relation between \mathbf{ad} and \mathbf{Ad} ?
Express the adjoint representation of $\mathfrak{sl}(2, \mathbb{R})$ in matrix form.
3. Define the *Killing form* of a Lie algebra.
Compute the Killing form of $\mathfrak{su}(2)$.
Sketch the proof that if G is a connected linear group, and the Killing form of LG is negative-definite, then G must be compact.
4. Sketch the representation theory of $\mathfrak{su}(3)$, defining carefully the terms *weight* and *weight vector*.
Determine the weights of the adjoint representation of $\mathfrak{su}(3)$.