

Finite Fields

Exercises on Chapter 2

Exercise 2

- * 1. What is the characteristic of the field \mathbb{R} ?
- ** 2. Show that the prime subfield of a field of characteristic 0 is \mathbb{Q} .
- ** 3. Find an infinite field of characteristic 2.
- ** 4. Show that an integral domain either has prime characteristic or else has characteristic 0.
- * 5. What is the characteristic of $\mathbb{Z}/(12)$?
- * 6. Show that every non-zero element in a ring of prime characteristic p has additive order p .
- ** 7. Does there exist a commutative ring of order 4 (ie with 4 elements) that is not a field?
- *** 8. Does there exist a non-commutative ring of order 6?
- *** 9. Find all commutative rings of order 12.
- ** 10. Show that a ring of characteristic n has an element of multiplicative order m for each factor m of n .