

Exercise 5

In Exercises 1–16 determine all solutions of the given congruence.

- * 1. $3x \equiv 1 \pmod{23}$
- * 2. $7x \equiv 1 \pmod{47}$
- ** 3. $5x \equiv 2 \pmod{210}$
- ** 4. $6x \equiv 7 \pmod{25}$
- ** 5. $8x \equiv 5 \pmod{31}$
- ** 6. $8x \equiv 12 \pmod{32}$
- ** 7. $12x \equiv 6 \pmod{21}$
- ** 8. $2x \equiv 2 \pmod{16}$
- ** 9. $20x \equiv 8 \pmod{24}$
- *** 10. $7x \equiv -3 \pmod{2009}$
- ** 11. $x^2 \equiv 1 \pmod{12}$
- ** 12. $x^2 \equiv -1 \pmod{15}$
- ** 13. $x^2 + x + 1 \equiv 0 \pmod{3}$
- ** 14. $x^2 - 2x + 3 \equiv 0 \pmod{5}$
- ** 15. $x^2 - 2 \equiv 0 \pmod{7}$
- *** 16. $x^4 + 2x^2 + x - 2 \equiv 0 \pmod{7}$
- * 17. What is the order of 10 in the additive group $\mathbb{Z}/(24)$?
- ** 18. Determine the orders of the elements 7, 11, 21 in the multiplicative group $(\mathbb{Z}/36)^\times$.
- ** 19. What is the order of the group $(\mathbb{Z}/36)^\times$?
- *** 20. Is the group $(\mathbb{Z}/36)^\times$ cyclic?

- *** 21. Is Christmas equally likely to take place on any day of the week?
- **** 22. Given integers $x_1, x_2, \dots, x_{11}$, show that there exists a finite sequence $a_1, \dots, a_{11}$ of numbers from $\{-1, 0, 1\}$ such that the sum

$$a_1x_1 + \dots + a_{11}x_{11}$$

is divisible by 2009.

- *** 23. Construct the field containing 4 elements.
- **** 24. Show that there is no field containing 6 elements.
- *** 25. Determine the orders of all the elements in $\mathbb{F}_{11}^\times$?
- ** 26. What is the order of the multiplicative group $\mathbb{F}_q^\times$?
- *** 27. How many elements are there of order 4 in $\mathbb{F}_{17}^\times$?
- *** 28. Prove that there is a multiple of 2009 which ends with the digits 000001.