Course 428

Elliptic Curves III

Dr Timothy Murphy

Maxwell Theatre Thursday, 4 May 2000 16:00–17:00

Attempt 3 questions. (If you attempt more, only the best 3 will be counted.) All questions carry the same number of marks.

1. Define the height $h(P)$ of a point on the elliptic curve

$$\mathcal{E}(\mathbb{Q}) : y^2 = x^3 + ax^2 + bx + c \qquad (a, b, c \in \mathbb{Z});$$

and show that there are only a finite number of points $P \in \mathcal{E}$ satisfying

$$h(P) \leq C$$

for any constant C.

State Mordell's Theorem; and explain the rôle of the height $h(P)$ in its proof.

2. Suppose $\mathcal{E}$ is the elliptic curve

$$\mathcal{E}(\mathbb{Q}) : y^2 = x^3 + ax^2 + bx \qquad (a, b \in \mathbb{Q}).$$

Outline the proof that $\mathcal{E}/2\mathcal{E}$ is finite.

3. Solve the equation

$$x^3 + 3x^2 + 1 = 0.$$

4. Show that the group on the elliptic curve

$$\mathcal{E}(\mathbb{Q}) : y^2 = x^3 - x$$

is finite, and find its order.