Course MA346H
Sample Exam Paper 1

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1. Explain carefully what you mean by a Turing machine, and show how such a machine defines a function

$$f : \mathbb{S} \to \mathbb{S} \cup \{\bot\},$$

where $\mathbb{S}$ is the set of finite strings of 0's and 1's, and $\bot$ denotes an undefined result.

Construct two Turing machines implementing the two functions

$$[m][n] \mapsto [m + n] \text{ and } [m][n] \mapsto [mn],$$

where

$$[m] = \underbrace{1 \ldots 1}_{m \text{ 1's}} 0.$$

2. Given sets X, Y, what is meant by saying that

   (a) $\#X = \#Y$,
   (b) $\#X \le \#Y$?

   Show that

   $$\#X \le \#Y \text{ and } \#Y \le \#Y \implies \#X = \#Y.$$

3. Define the *algorithmic entropy* $H(s)$ of a string s (of 0's and 1's).

   Show that
   $$H(s) \le |s| + H(|s|) + O(1).$$

   Show conversely that there exists a constant C such that for each $n \in \mathbb{N}$ there exists a string s of length n such that

   $$H(s) \ge n + H(n) - C.$$