Assignment for course MA346H

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1 Assignment format

1. Chaitin's version of the Turing Machine must be used, with action-set
$$A = \{\texttt{Noop}, \texttt{Swap}, \texttt{Left}, \texttt{Right}, \texttt{Read}, \texttt{Write}\}.$$
2. The rules of the Turing Machine must be specified as a sequence of quadruples (m, b, a, n), with $m, n \in N$, $b \in 0, 1$ and $a \in A$, eg $(2, 1, \texttt{Left}, 3)$, meaning $(q_2, 1) \to (\texttt{Left}, q_3)$.
3. The machine should be specified as a sequence of rules, with one rule on each line.
4. Initial and final states are both 0.
5. Not all rules need be given. Missing rules are assumed to output $(\texttt{Noop}, 0)$, ie the machine will halt.
6. The tape need not be 'clean', ie containing only 0's, when the machine halts.
7. The numbers $n \in \mathbb{N}$ should be encoded for input to and output from the machine in the form
$$\langle n \rangle = \overbrace{1 \cdots 1}^{n} 0.$$
The n-tuple $(m_1, \ldots, m_n)$ should be encoded as
$$\langle m_1 \rangle \langle m_2 \rangle \cdots \langle m_n \rangle.$$
The string $s = b(1)b(2) \cdots b(n) \in \mathbb{S}$ should be encoded in the form
$$\langle s \rangle = 1b(1)1b(2)1 \cdots 1b(n)00.$$
Predicates should return either 0 (false) or 1 (true).

2 Functions and Predicates

1. $f(m,n) = m^n$.
2. $f(n) = n^n$.
3. $f(n) = n!$.
4. $P(n) \equiv n$ is prime.
5. $P(n) \equiv n$ is a perfect square.
6. $f(m,n) = m \bmod n$.
7. $f(m,n) = \gcd(m,n)$.
8. $f(m,n) = \mathrm{lcm}(m,n)$.
9. $f(n) = [\sqrt{n}]$, ie the largest integer r such that $r^2 \leq n$..
10. $P(m,n) \equiv m$ divides n.
11. $P(m,n) \equiv m$ is less than n. .
12. $P(m,n) \equiv m,n$ are coprime.
13. $f(n) = F(n)$, the nth Fibonacci number.
14. $f(n) = B(n)$, the binary code for n.
15. $f(m,n) = \binom{m}{n}$, the coefficient of x^m in $(1+x)^n$.
16. $f(m,n) = |n-m|$.
17. $f(s) = ss$, the string s doubled.
18. $f(s) = s^*$, the string s reversed.
19. $P(m, n_1, \ldots, n_r) \equiv m \in \{n_1, \ldots, n_r\}$.
20. $f(m,n) = [m/n]$, ie the largest integer r such that $r \leq m/n$.
21. $f(m,n) = \max(m,n)$.
22. $f(m,n) = \min(m,n)$.
23. $f(n_1, \ldots, n_r) = \sum n_i$.