



Course 424

Group Representations II

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School of Mathematics Tuesday, 30 April 1991 16:00–18:00

Answer as many questions as you can; all carry the same number of marks.

All representations are finite-dimensional over \mathbb{C} .

1. Define a *measure* on a compact space. State carefully, and outline the main steps in the proof of, Haar's Theorem on the existence of an invariant measure on a compact group.

Prove that every representation of a compact group is semisimple.

2. Which of the following groups are (a) compact, (b) connected:

$\mathbf{O}(n)$, $\mathbf{SO}(n)$, $\mathbf{U}(n)$, $\mathbf{SU}(n)$, $\mathbf{GL}(n, \mathbb{R})$, $\mathbf{SL}(n, \mathbb{R})$, $\mathbf{GL}(n, \mathbb{C})$, $\mathbf{SL}(n, \mathbb{C})$?

(Note: You must justify your answer in each case; no marks will be given for unsupported assertions)

3. Determine the conjugacy classes in $\mathbf{SU}(2)$; and prove that this group has just one simple representation of each dimension. Find the character of this representation.

Determine the representation-ring of $\mathbf{SU}(2)$.

4. Show that there exists a surjective homomorphism

$$\Theta : \mathbf{SU}(2) \rightarrow \mathbf{SO}(3)$$

having finite kernel.

Hence or otherwise determine all simple representations of $\mathbf{SO}(3)$.

Determine also all simple representations of $\mathbf{O}(3)$.