

Course 424 Group Representations II

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G.M.B. Tuesday, 13 April 1993 14:00–16:00

Answer as many questions as you can; all carry the same number of marks.

Unless otherwise stated, all representations are finite-dimensional over \mathbb{C} .

1. Define a *measure* on a compact space. State carefully, and outline the main steps in the proof of, Haar's Theorem on the existence of an invariant measure on a compact group.

Prove that every representation of a compact group is semisimple.

2. Which of the following groups are (a) compact, (b) connected:

 $\mathbf{SO}(2), \mathbf{O}(2), \mathbf{U}(2), \mathbf{SU}(2), \mathbf{Sp}(2), \mathbf{GL}(2, \mathbb{R}), \mathbf{SL}(2, \mathbb{R}), \mathbf{GL}(2, \mathbb{C}), \mathbf{SL}(2, \mathbb{C}), \mathbb{T}^2 ?$

(Note: Justify your answer in each case; no marks will be given for unsupported assertions)

3. Determine the conjugacy classes in SU(2); and prove that this group has just one simple representation of each dimension.

Find the character of the representation D(j) of dimensions 2j + 1 (where $j = 0, \frac{1}{2}, 1, \frac{3}{2}, \ldots$).

Express each product D(i)D(j) as a sum of simple representations D(k).

4. Show that there exists a surjective homomorphism

$$\Theta: \mathbf{SU}(2) \to \mathbf{SO}(3)$$

with finite kernel.

Hence or otherwise determine all simple representations of SO(3).

Determine also all simple representations of O(3).

5. Explain the division of simple representations of a finite or compact group G into real, essentially complex and quaternionic.

Determine into which of these 3 categories the representations D(j) of $\mathbf{SU}(2)$ fall.