Course 375
Information Theory

Sample Exam

1991

Answer as many questions as you can; all carry the same number of marks.

1. Define the *entropy*
$$H(X) = H(p_1, \ldots, p_n)$$
of a finite probability space X; and show that the entropy is maximised for given n when the probabilities are equal. When is it minimised?

2. Show that the joint entropy $H(XY)$ of 2 finite (but not necessarily independent) probability spaces X, Y satisfies
$$H(XY) \leq H(X) + H(Y).$$

3. Define the *algorithmic entropy* $H(s)$ of a string s. Extending the definition to the entropy $H(n)$ of a natural number n, show that
$$H(s) \leq \|s\| + H(\|s\|) + O(1),$$
where $\|s\|$ denotes the length of the string s.

   Sketch the proof that this is (in a sense to be defined) the best possible result.

4. What is meant by saying that a set $S = \{s_i\}$ of strings is *prefix-free*? Show that if the set $\{n_i\}$ of natural numbers satisfies
$$\sum_i 2^{-n_i} \leq 1$$

then a prefix-free set $\{s_i\}$ of strings can be found such that

$$\|s_i\| \le n_i.$$

5. Define the *statistical algorithmic entropy* $h(s)$ of a string s; and show that

$$h(s) = H(s) + O(1).$$

6. Define the *joint entropy* $H(s, t)$ of 2 strings s, t, and the *relative entropy* $H(s \mid t)$ of s given t; and show that

$$H(s, t) = H(s) + H(t \mid s) + O(1).$$