Course 424

Group Representations II

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School of Mathematics Tuesday, 30 April 1991 16:00–18:00

Answer as many questions as you can; all carry the same number of marks.
All representations are finite-dimensional over $\mathbb{C}$.

1. Define a *measure* on a compact space. State carefully, and outline the main steps in the proof of, Haar's Theorem on the existence of an invariant measure on a compact group.

   Prove that every representation of a compact group is semisimple.

2. Which of the following groups are (a) compact, (b) connected:

   $$\mathbf{O}(n), \mathbf{SO}(n), \mathbf{U}(n), \mathbf{SU}(n), \mathbf{GL}(n, \mathbb{R}), \mathbf{SL}(n, \mathbb{R}), \mathbf{GL}(n, \mathbb{C}), \mathbf{SL}(n, \mathbb{C})?$$

   (Note: You must justify your answer in each case; no marks will be given for unsupported assertions)

3. Determine the conjugacy classes in $\mathbf{SU}(2)$; and prove that this group has just one simple representation of each dimension. Find the character of this representation.

   Determine the representation-ring of $\mathbf{SU}(2)$.

4. Show that there exists a surjective homomorphism

   $$\Theta : \mathbf{SU}(2) \to \mathbf{SO}(3)$$

   having finite kernel.

   Hence or otherwise determine all simple representations of $\mathbf{SO}(3)$.

   Determine also all simple representations of $\mathbf{O}(3)$.