Course 424

Group Representations IIIa

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Arts Block A2039 Friday, 3 February 1989 15.00–17.00

Answer as many questions as you can; all carry the same number of marks.

Unless otherwise stated, all Lie algebras are over $\mathbb{R}$, and all representations are finite-dimensional over $\mathbb{C}$.

1. Define a *Lie algebra* L, and a *representation* of Ł.

   Explain how a representation α of a linear group G gives rise to a representation of its Lie algebra ŁG.

   Show that if 2 Lie algebras have the same complexification then their representations correspond. Hence or otherwise show that all the representations of $\mathbf{sl}(\mathbf{2}, \mathbb{R})$ are semisimple.

2. Define the *adjoint represntation* **ad** of a Lie algebra Ł (verifying that it is indeed a representation).

   Define also the adjoint representation **Ad** of the linear group G in ŁG. What is the relation between **ad** and **Ad**?

   Express the adjoint representation of $\mathbf{sl}(\mathbf{2}, \mathbf{R})$ in matrix form.

3. Define the *Killing form* of a Lie algebra.

   Compute the Killing form of $\mathbf{su}(\mathbf{2})$.

   Sketch the proof that if G is a connected linear group, and the Killing form of ŁG is negative-definite, then G must be compact.

4. Sketch the representation theory of $\mathbf{su}(\mathbf{3})$, defining carefully the terms *weight* and *weight vector*.

   Determine the weights of the adjoint representation of $\mathbf{su}(\mathbf{3})$.