



Course 424

Group Representations II

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Joly Theatre Friday, 6 April 2001 16:00–17:30

*Attempt 5 questions. (If you attempt more, only the best 5 will be counted.) All questions carry the same number of marks.
All representations are finite-dimensional over $\mathbb{C}$.*

1. Define a *measure* on a compact space. State carefully, and outline the main steps in the proof of, Haar's Theorem on the existence of an invariant measure on a compact group.
2. Which of the following groups are (a) compact, (b) connected:
 $O(n)$, $SO(n)$, $U(n)$, $SU(n)$, $GL(n, \mathbb{R})$, $SL(n, \mathbb{R})$, $GL(n, \mathbb{C})$, $SL(n, \mathbb{C})$?
(Justify your answer in each case.)
3. Prove that every representation of a compact group is semisimple.
Give an example of a representation of the additive group $\mathbb{R}$ which is not semisimple.
4. Prove that every simple representation of a compact *abelian* group is 1-dimensional.
Determine the simple representations of $U(1)$.
5. Determine the conjugacy classes in $SU(2)$.
Prove that $SU(2)$ has just one simple representation of each dimension $1, 2, \dots$; and determine the character of this representation.
6. Determine the conjugacy classes in $SO(3)$.
Prove that $SO(3)$ has just one simple representation of each odd dimension $1, 3, 5, \dots$.