MA1M01 Calculus Assignment 7

Michælmas term week 9

www.maths.tcd.ie/pub/MA1M01/Calculus/

1. **[40 points]** Use the formula $\int udv = uv - \int vdu$:

a. Let $u = x$ and $dv = \cos(x)$.

Then $\frac{du}{dx} = 1$ and $du = dx$. As $dv = \cos(x)$ and $v = \int dv$,

$v = \int \cos(x)dx = \sin(x)$.

$\int udv = uv - \int vdu \implies \int x\ \cos(x)dx = x\sin(x) - \int \sin(x)dx$

$= x\sin(x) - (-\cos(x))$

$= x\sin(x) + \cos(x) + c$.

b. Let $u = x$ and $dv = \cos(5x)$.

Then $\frac{du}{dx} = 1$ and $du = dx$. As $dv = \cos(x)$ and $v = \int dv$,

$v = \int \cos(5x)dx = \frac{\sin(5x)}{5}$ (use substitution to get this integral).

$\int udv = uv - \int vdu \implies \int x\ \cos(5x)dx = \frac{x\sin(5x)}{5} - \int \frac{\sin(5x)}{5}dx$

$= \frac{x\sin(5x)}{5} - \frac{1}{5}(-\frac{\cos(5x)}{5}) + c$

$= \frac{x\sin(5x)}{5} + \frac{\cos(5x)}{25} + c$

c. Let $u = x$ and $dv = e^x$.

Then $\frac{du}{dx} = 1$ and $du = dx$. As $dv = e^x$ and $v = \int dv$,

$v = \int e^x dx = e^x$.

$\int udv = uv - \int vdu \implies \int xe^x dx = x(e^x) - \int (e^x)dx$

$= x(e^x) - (e^x) + c.$

d. Let $u = x$ and $dv = e^{-x}$.

Then $\frac{du}{dx} = 1$ and $du = dx$. As $dv = e^{-x}$ and $v = \int dv$,

$v = \int e^{-x}dx = -e^{-x}$ (do this using substitution).

$\int udv = uv - \int vdu \implies \int xe^{-x}dx = x(-e^{-x}) - \int(-e^{-x})dx$

$= -x(e^{-x}) - (e^{-x}) + c.$

2. **[10 points]**

a. $\ln(2x^3) - \ln(x) = \ln(16) + \ln(x)$

$\implies \ln(\frac{2x^3}{x}) = \ln(16x)$

$\implies \frac{2x^3}{x} = 16x$

$\implies 2x^2 = 16x \implies x = 8.$

b. $\ln(2x^2) + \ln(x) = \ln(32) - \ln(x)$

$\implies \ln(2x^3) = \ln(\frac{32}{x})$

$\implies 2x^3 = \frac{32}{x}$

$\implies 2x^4 = 32 \implies x = 2.$

3. **[30 points]**

a. $\frac{\sqrt{1+\frac{x^2}{c}}(1)-x(\frac{1}{2}(1+\frac{x^2}{c})^{-\frac{1}{2}}\frac{1}{c}2x}{1+\frac{x^2}{c}}$

$$= \frac{\sqrt{1+\frac{x^2}{c}}-x\left(\frac{2x}{2c\sqrt{1+\frac{x^2}{c}}}\right)}{1+\frac{x^2}{c}}$$

$$= \frac{\sqrt{1+\frac{x^2}{c}}-\left(\frac{x^2}{c\sqrt{1+\frac{x^2}{c}}}\right)}{1+\frac{x^2}{c}}$$

$= \frac{1}{\left(1+\frac{x^2}{c}\right)^{\frac{3}{2}}}.$

b. $(3x^2)e^{x^3}$

c. $\frac{x^2(3x^2)(e^{x^3})-e^{x^3}(2x)}{x^4}$

$= \frac{3x^4(e^{x^3})-e^{x^3}(2x)}{x^4}$

4. **[20 points]**

a. $p(0) = (210)^5 e^{(0.1)(0)}$

$\Longrightarrow p(0) = (210)^5.$

b. We need to find t such that $p(t) = 2 \times p(0)$.

Therefore $(210^5)e^{0.1t} = 2 \times (210^5)$

$\Longrightarrow 2 = e^{0.1t}$

$\Longrightarrow \ln(2) = \ln(e^{0.1t})$

$\Longrightarrow \ln(2) = 0.1t$

$\Longrightarrow t = 6.9317.$

c. $\frac{dp}{dx} = (210)^5(0.1)e^{0.1t}.$

After 24 hours the instantaneous rate of change is given by

$(210)^5(0.1)e^{0.1(24)} = (210)^5(1.1023).$

Homework is due one week from when it is given in the tutorial you are assigned to. This set should be handed up in week 10.