MA1M01 Calculus Assignment 6

Michaelmas term week 9

www.maths.tcd.ie/pub/MA1M01/Calculus/

1. **[20 points]** Use the product rule to determine the derivatives of both of the following:

   (a) **[5 points]** Using the product rule first then the chain rule for the derivative of the trigonometric function:

$$\frac{dy}{dx} = \left(\frac{d}{dx}x^2\right)\sin(3x) + x^2\left(\frac{d}{dx}\sin(3x)\right)$$

$$= 2x\sin(3x) + 3x^2\cos(3x)$$

   (b) **[5 points]** Using the product rule first and then the chain rule for the derivative of the trigonometric function:

$$\frac{dy}{dx} = \frac{3}{8}\left(\left(\frac{d}{dx}x\right)\cos(7x^3 - x) + x\left(\frac{d}{dx}\cos(7x^3 - x)\right)\right)$$

$$= \frac{3}{8}\left(\cos(7x^3 - x) - x(21x^2 - 1)\sin(7x^3 - x)\right)$$

   (c) **[5 points]** Using the product rule and the chain rule:

$$\frac{dy}{dx} = \left(\frac{d}{dx}6x\right)(9x - 3)^3 + 6x\left(\frac{d}{dx}(9x - 3)^3\right)$$

$$= 6(9x - 3)^3 + 162x(9x - 3)^2$$

   (d) **[5 points]** Using the product rule and the chain rule:

$$\frac{dy}{dx} = \left(\frac{d}{dx}(2x^2 - 7)^4\right)(3x^4 - 1)^2 + (2x^2 - 7)^4\left(\frac{d}{dx}(3x^4 - 1)^2\right)$$

$$= 16x(2x^2 - 7)^3(3x^4 - 1)^2 + 24x^3(3x^4 - 1)(2x^2 - 7)^4$$

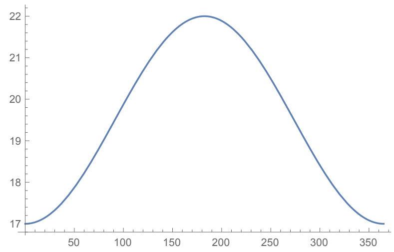
2. **[20 points]** The time of sunset over the course of the year can be approxi-

mated by $s(t)$

$$s(t) = 19:30 - 2:30\cos(\frac{2\pi t}{365})$$

where t represents time in days, starting from $t = 0$ on January 1st.

(a) **[10 points]**



(b) **[10 points]** $s(11) = 19:30 - 2:30\cos(\frac{2\pi(11)}{365}) = 19:30 - 2:30(0.982) \simeq 17:03$

3. **[40 points]** Integrate each of the following, using substitution where appropriate.

(a) **[10 points]** This can be done by making the substitution $u = 6x$ and changing to an integral of the form

$$\int f(u)\ du$$

To do this need to find the relationship between du and dx which is done by examining the derivative of u with respect to x, which gives

$$\frac{du}{dx} = 6 \Longrightarrow \frac{du}{6} = dx$$

So the integral has now become

$$\frac{1}{6}\int \cos(u)\ du = \frac{1}{6}\sin(u) + c = \frac{1}{6}\sin(6x) + c$$

(b) **[10 points]** This integral is performed by making the substitution $u = 9x^2$ and recasting it as an integral of $f(u)$, as before

$$\frac{du}{dx} = 18x \Longrightarrow \frac{du}{18} = xdx$$

So the integral becomes

$$\frac{4}{18}\int \sin(u)\ du = \frac{-2}{9}\cos(u) + c = \frac{-2}{9}\cos(9x^2) + c$$

(c) **[10 points]** This integral can be performed by making either the substitution $u = \sin(x)$ or $u = \cos(x)$, moving forward with the former:

$$\frac{du}{dx} = \cos(x) \Longrightarrow du = \cos(x)dx$$

Which gives

$$\int u\ du = \frac{u^2}{2} + c = \frac{1}{2}\sin^2(x) + c$$

If the other substitution is made the answer obtained is $-\frac{1}{2}\cos^2(x) + c$, this is not a problem since they are the same up to a constant, this can be seen from identity $\cos^2(x) + \sin^2(x) = 1$.

(d) **[10 points]** This can be done by making the substitution $u = \frac{7}{9x^2}$ or by tidying up the integrand I will do the latter:

$$\int \frac{3}{x^3}\Big(\frac{7}{9x^2}\Big)^{\frac{3}{2}}\ dx$$

$$= \int \frac{3}{x^3}\Big(\frac{7}{9}\Big)^{\frac{3}{2}}\frac{1}{x^3}\ dx$$

$$= 3\Big(\frac{7}{9}\Big)^{\frac{3}{2}}\int \frac{1}{x^6}\ dx$$

$$= -\frac{3}{5}\Big(\frac{7}{9}\Big)^{\frac{3}{2}}\frac{1}{x^5} + c$$

4. **[20 points]** Use the quotient rule to evaluate the derivative of $y = \frac{\cos(x)}{\sin^2(x)}$

$$\frac{dy}{dx} = \frac{(\sin^2(x))(\frac{d}{dx}\cos(x)) - (\cos(x))(\frac{d}{dx}\sin^2(x))}{\sin^4(x)}$$

$$= \frac{-\sin^3(x) - 2\cos^2(x)\sin(x)}{\sin^4(x)}$$

$$= \frac{-\sin(x)\big(\sin^2(x) + \cos^2(x) + \cos^2(x)\big)}{\sin^4(x)}$$

$$= \frac{-(1 + \cos^2(x))}{\sin^3(x)}$$

Homework is due one week from when it is given in the tutorial you are assigned to. This set should be handed up in week 10.