MA1M01 Calculus Assignment 1 Michælmas term week 3

www.maths.tcd.ie/pub/MA1M01/Calculus/

1. [10 points] Convert the following angles:

(a) 6.28 radians	(b) 0.78 radians
(c) -143.2 degrees	(d) 154.7 degrees

2. [20 points] The points (1,0), (0,1) lie on the unit circle in the xy-plane. Find a third point, (x, y), which also lies on the unit circle and is equidistant from these two points.

As points on the unit circle at an angle θ are given by $x = \cos(\theta)$, $y = \sin(\theta)$, one such point is

$$(x,y) = \left(\cos\left(\frac{\pi}{4}\right), \sin\left(\frac{\pi}{4}\right)\right) = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right).$$

The other point is at the opposite side of the circle, at an angle of $\theta = \frac{5\pi}{4}$,

 $(x,y) = \left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right).$



Another way to approach this is to notice that y = x, so we need a number x such that $2x^2 = 1$ (so as to satisfy the equation of the unit circle). The two solutions to this are $x = \pm \frac{1}{\sqrt{2}}$.

- 3. [20 points] Differentiate the following functions with respect to θ .
 - (a) $\cos(\theta), \frac{d}{d\theta}\cos(\theta) = -\sin(\theta)$
 - (b) $\sin(\theta), \frac{d}{d\theta}\sin(\theta) = \cos(\theta)$
 - (c) $A\cos(\theta) + B\sin(\theta), \frac{d}{d\theta} (A\cos(\theta) + B\sin(\theta)) = -A\sin(\theta) + B\cos(\theta)$
 - (d) $\sin(5\theta + 2\pi), \frac{d}{d\theta}\sin(5\theta + 2\pi) = \frac{d}{d\theta}\sin(5\theta) = \frac{d}{du}\sin(u)\frac{du}{d\theta} = 5\cos(5\theta)$
- 4. [50 points]
 - (a) $20t\sin(2t^2)$

(b) $2x + 2\cos(2x + \pi)$ at $x = \pi$ is $2\pi - 2$ (c) $-6x\sin(3x^2 + 5)$ (d) $\cos(x^{-1} - \cos(x))(-x^{-2} + \sin(x))$ (e) $-\sin(t^3 + \sin(3t + \pi))(3t^2 + 3\cos(3t + \pi))$