

MA1M01 Calculus Assignment 3

Michælmas term week 5

www.maths.tcd.ie/pub/MA1M01/Calculus/

1. [40 points] Find the following indefinite integrals:

a. $\int 5x^6 dx = 5\frac{x^7}{7} + c$

b. $\int \frac{4x^5 - 3x}{7} dx = \frac{1}{7} \left(\frac{2x^6}{3} - \frac{3x^2}{2} \right) + c$

c. $\int 3x^7 + 7x^2 + 4x + 3 dx = \frac{3x^8}{8} + \frac{7x^3}{3} + 2x^2 + 3x + c$

d. $\int x^{\frac{2}{3}} dx = \frac{3}{5} x^{\frac{5}{3}} + c$

e. $\int 3x^{-5} dx = \frac{-3}{4} x^{-4} + c$

f. $\int \frac{-3x^{\frac{2}{3}} + 7x^{-2}}{8} dx = \frac{1}{8} \left(\frac{-3x^{\frac{5}{3}}}{\frac{5}{3}} + \frac{7x^{-1}}{-1} \right) + c = -\frac{9x^{\frac{8}{3}} + 35}{40x} + c$

2. [40 points] Find the following definite integrals:

a. $\int_{-a}^a x dx = \left[\frac{x^2}{2} \right]_{-a}^a = 0$

b. $\int_{-1}^1 x^2 + 4x - 3 dx = \left[\frac{x^3}{3} + \frac{4x^2}{2} - 3x \right]_{-1}^1 = \frac{-16}{3}$

c. $\int_{-4}^{-1} \frac{1}{x^3} dx = \left[\frac{x^{-2}}{-2} \right]_{-4}^{-1} = \frac{-15}{32}$

d. $\int_{-1}^1 x^7 + \frac{2}{3}x dx = \left[\frac{x^8}{8} + \frac{2}{3} \frac{x^2}{2} \right]_{-1}^1 = 0$

e. $\int_0^4 x^{\frac{-1}{5}} dx = \left[\frac{x^{\frac{4}{5}}}{\frac{4}{5}} \right]_0^4 = \frac{5}{2^{\frac{2}{5}}}$

f. $\int_{-2}^3 \frac{4(x+1)^2 + x^7}{2} dx$
 $= \int_{-2}^3 \frac{4x^2 + 4 + 8x + x^7}{2} dx$
 $= \frac{1}{2} \left[\frac{4x^3}{3} + 4x + \frac{8x^2}{2} + \frac{x^8}{8} \right]_{-2}^3 = \frac{20995}{48}$

3. [5 points] $\frac{f(3)-f(-3)}{3-(-3)} = \frac{16-(-2)}{6} = 3$

4. [5 points] $\frac{df}{dx} = \frac{-3}{2} x^{\frac{1}{2}} + 4$.

So rate of change at $x = 16$ is $\frac{-3}{2} (16)^{\frac{1}{2}} + 4 = -2$

5. [10 points]

$$(a) \frac{1}{1-(-3)} \int_{-3}^1 x^3 + 2dx = \frac{1}{4} \left[\frac{x^4}{4} + 2x \right]_{-3}^1 = -3$$

$$(b) \frac{1}{1-(-3)} \int_{-3}^1 3x^3 + 7x^2 + 1dx = \frac{1}{4} \left[3\frac{x^4}{4} + 7\frac{x^3}{3} + x \right]_{-3}^1 = \frac{28}{12}$$

*Homework is due one week from when it is given in the tutorial you are assigned to.
This set should be handed up in week 6.*