MA1M01 Calculus Assignment 2 Michaelmas term week 4

www.maths.tcd.ie/pub/MA1M01/Calculus/

- 1. [20 points] Solve each of the following for x such that f(x) = 0:
 - (a) [5 points]Since this is a product, one of the terms must be zero which implies that either

$$x = 0$$
 or $(x - 3) = 0 \Longrightarrow x = 0, 3$

Using the quadratic formula with a = 1, b = -3 and c = 0 gives the same result.

(b) **[5 points**]

$$x^{2} - 1 = 0$$
$$\implies (x - 1)(x + 1) = 0$$
$$\implies x = 1, -1$$

Using the quadratic formula with a = 1, b = 0 and c = -1 gives the same result.

(c) **[5 points]**

$$x^{2} - 6x + 8 = 0$$
$$\implies (x - 4)(x - 2) = 0$$
$$\implies x = 2, 4$$

Using the quadratic formula with a = 1, b = -6 and c = 8 gives the same result.

(d) **[5 points]**

$$x^2 + 10x + 21 = 0$$

$$\implies (x+7)(x+3) = 0$$
$$\implies x = -7, -3$$

Using the quadratic formula with a = 1, b = 10 and c = 21 gives the same result.

2. [20 points] A ball is dropped from a height of 125 metres. The height of the ball above ground level follows the equation

$$h(t) = 125 - 5t^2$$

where t represents time in seconds and $t \ge 0$.

(a) **[10 points]**

$$h(2) = 125 - 5(2)^2 = 125 - 5(4)$$

= 125 - 20 = 105 metres

(b) [10 points]

$$h(t) = 5(25 - t^2) = 5(t + 5)(t - 5) \Longrightarrow t = 5$$
 seconds

since $t \ge 0$

(a) [40 points] Compute the derivative $\frac{dy}{dx}$ of each of the following

$$y = x^2 + x \Longrightarrow \frac{dy}{dx} = 2x + 1$$

(b) [10 points]

$$y = 6x(x-1) = 6x^2 - 6x \Longrightarrow \frac{dy}{dx} = 12x - 6$$

(c) **[10 points]**

$$y = (x+2)^3 \Longrightarrow \frac{dy}{dx} = 3(x+2)^2 \times \frac{d}{dx}(x+2) = 3(x+2)^2(1)$$

(d) [**10 points**]

$$y = \frac{x^4 - 3x^2}{7} \Longrightarrow \frac{dy}{dx} = \frac{1}{7} \times (4x^3 - 6x) = \frac{4x^3 - 6x}{7}$$

- 3. [20 points] Compute the slope of the tangent line to y at the given point for both of the following:
 - (a) **[10 points]**

$$y = x^3 - 8x^2 \Longrightarrow \frac{dy}{dx} = 3x^2 - 16x$$

This is then evaluated at x = 2 to give

$$\left. \frac{dy}{dx} \right|_{x=2} = 3(2)^2 - 16(2) = 3(4) - 32 = 12 - 32 = -20$$

(b) **[10 points]**

$$y = -\frac{3x^4}{4} + \frac{4x}{9} \Longrightarrow \frac{dy}{dx} = -3x^3 + \frac{4}{9}$$

This is then evaluated at x = -1 to give

$$\frac{dy}{dx}\Big|_{x=-1} = -3(-1)^3 + \frac{4}{9} = 3 + \frac{4}{9} = \frac{27}{9} + \frac{4}{9} = \frac{31}{9}$$

Homework is due one week from when it is given in the tutorial you are assigned to. This set should be handed up in week 5.