MA1M01 Calculus Assignment 8 Michælmas term week 10

www.maths.tcd.ie/pub/MA1M01/Calculus/

1. [50 points] Carbon–14 (¹⁴C) is a radioactive isotope of carbon and has a half-life of $t_{\frac{1}{2}} \approx 5730$ years. Beta decay can be modelled as

$$N(t) = ce^{-\lambda t}$$

where N(t) is the number of atoms of ¹⁴C at time t (measured in years).

- (a) What is the physical meaning of the constant c? (*Hint: with questions that look like this and 2b, try setting* t = 0.)
- (b) Find the exponential decay rate λ .
- (c) You find yourself in Hamunaptra (the lost city of the dead) and discover canopic jars carrying Anck–su–Namun's preserved organs. If $\frac{1}{3}$ of the ¹⁴C is gone, how old are these organs?
- (d) A band of American treasure hunters want to sell you the Book of the Dead. If 0.5% of the ¹⁴C is lost, how old is it?
- 2. [50 points] Newton's law of cooling states that

$$\frac{d}{dt}T(t) = -k(T(t) - T_a),$$

where T(t) is the temperature of the object at time t (measured in hours), T_a is the ambient temperature and k is some constant of proportionality.

- (a) Show that $T(t) = ce^{-kt} + T_a$ is a solution to the above equation. (*Hint: just differentiate, it's really easy.*)
- (b) What is the physical meaning of the constant c?
- (c) You find the body of High Priest Imhotep at time t_0 after his death at a temperature of $T(t_0) = 26^{\circ}$ C. One hour later, his temperature is measured to be $T(t_0 + 1) = 25^{\circ}$ C. If the ambient temperature of the room is 18°C, what is the value of the constant k?
- (d) How long has Imhotep been dead for? I.e. what is the value of t_0 ? Assume that Imhotep has a normal body temperature of 37°C.

Homework is due one week from when it is given in the tutorial you are assigned to. This set should be handed up in week 11.