

**THE CATECHISM OF THE
AUTHOR OF THE MINUTE PHILOSOPHER
FULLY ANSWERED**

By

Jacob Walton

Edited by David R. Wilkins

2002

NOTE ON THE TEXT

The following spellings, differing from modern British English, are employed in the original 1735 edition: , expresly,

David R. Wilkins
Dublin, June 2002

THE
CATECHISM
OF THE
AUTHOR
OF THE
MINUTE PHILOSOPHER
FULLY ANSWER'D.

By *J. WALTON*.

—*Neque enim lex æquior ulla*
Quam necis Artifices Arte perire sua.

Ovid.

DUBLIN:

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For WILLIAM SMITH at the *Hercules*,
Bookseller, in *Dame-street*, 1735.

THE CATECHISM, &c.

I. In the Appendix of the *Defence of Free-thinking in Mathematics*, the Author, out of his greatly benevolent and truly Christian Spirit, has composed a CATECHISM which he recommends to my Scholars: this CATECHISM with its Introduction I shall transcribe in his own Words, and fully and distinctly answer the several Questions contained therein.

The INTRODUCTION.

II. “This Vindicator, indeed, by his dissembling nine Parts in ten of the Difficulties proposed in the *Analyst*, sheweth no Inclination to be CATECHISED by me. But his Scholars have a Right to be informed. I therefore, recommend it to them, not to be imposed on by hard Words and magisterial Assertions, but carefully to pry into his Sense, and sift his Meaning, and particularly to insist on a distinct Answer to the following Questions.”

The CATECHISM.

“Let them ask him, whether he can conceive Velocity without Motion, or Motion without Extension, or Extension without Magnitude? If he answers that he can, let him teach them to do the same. If he cannot, let him be asked, how he reconciles the Idea of a Fluxion which he gives (P. 13.) with common Sense? Again, let him be asked, whether nothing be not the Product of nothing multiply’d by something? And if so, when the Difference between the Gnomon and the Sum of the Rectangles vanisheth, whether the Rectangles themselves do not also vanish? that is, when ab is nothing, whether $Ab + Ba$ be not also nothing? that is, whether the Momentum of AB be not nothing? Let him then be asked, what his Momentums are good for, when they are thus brought to nothing? Again, I wish he were asked to explain the Difference between a Magnitude infinitely small and a Magnitude infinitely diminish’d? If he saith there is no Difference, then let him be asked, how he DARES to explain the Method of Fluxions by the *Ratio* of Magnitudes infinitely diminish’d (P. 9.) when Sir *Isaac Newton* hath expresly excluded all Consideration of Quantities infinitely small? If this able Vindicator should say that Quantities infinitely diminish’d are nothing at all, and consequently that, according to him, the first and last *Ratios* are Proportions between nothings, let him be desired to make Sense of this, or explain what he means by *Proportions between nothings*. If he should say the ultimate Proportions are the Ratios of mere Limits, then let him be asked how the Limits of Lines can be proportioned or divided? After all, who knows but this Gentleman, who hath already complained of me for an uncommon Way of treating Mathematics and Mathematicians, may (as well as the *Cantabrigian*) cry out *Spain* and the *Inquisition* when he finds himself thus closely pursued and beset with Interrogatories? That we may not therefore seem too hard on an innocent Man, who probably meant nothing, but

was betray'd by following another into Difficulties and Straits that he was not aware of, I shall propose one simple Expedient, by which his Disciples (whom it most concerns) may soon satisfy themselves, whether this Vindicator really understands what he takes upon him to vindicate. It is in short, that they would ask him to explain the second, third or fourth Fluxions upon his Principles. Be this the Touchstone of his Vindication. If he can do it, I shall own my self much mistaken: If he cannot, it will be evident that he was much mistaken in himself, when he presumed to defend Fluxions without so much as knowing what they are. So having put the merits of the Cause on this Issue, I leave him to be tried by his Scholars."

III. In this CATECHISM I am first to be asked, "Whether I can conceive Velocity without Motion, or Motion without Extension, or Extension without Magnitude?" These Questions are more clearly express'd in the 29th and 30th Queries of his *Analyst*, where he asks, "Whether we can form an Idea or Notion of Velocity distinct from and exclusive of its Measures? Whether Motion can be conceived in a Point of Space? And if Motion cannot, whether Velocity can? And if not, whether a first or last Velocity can be conceived in a mere Limit, either initial or final, of the described Space?" I answer, I can conceive Velocity and Motion in a *Point of Space*; that is without any assignable Length or Extension described by it, and so might he too if he had understood and consider'd the Nature of Motion. For Motion is an Effect of some Cause acting on the Thing moved; which Effect, setting aside all Resistance, will ever be proportional to the whole Action of the generating Cause: And therefore if a Cause acts continually upon a given Thing without any Interruption, there must be a continual Increase of its Velocity: The Velocity cannot be the same in any two different Points of the Space described, however near those Points may be to each other. For if it was, there must be a Cessation of the Action of the moving Cause during the Passage of the Thing thro' the Space comprehended between the two Points; which is contrary to the Supposition.



If the Thing moved be a Point, and the Space described by this Point setting out from a State of Rest at A, be A D; and if the Point be acted upon continually by a moving Cause during the Time of its going from A to D, and the Cause act continually with the same Degree of Strength during that Time; the Velocity will continually increase with the Time of the Motion, or with the Square Root of the Space described; that is, the Velocities in the Points C and D, will be in the same Proportion as the Times of describing A C and A D, or as the Square Roots of A C and A D. These are the Laws of Motion observed by Bodies falling from small Heights to the Surface of the Earth, setting aside the Resistance of the Air, and supposing the Bodies to have exactly the same Weights during the Times of their Fall.

If the Cause acts continually upon the moving Point, but with different Degrees of Strength during the Time of the Motion; the Velocity will not increase with the Time, nor

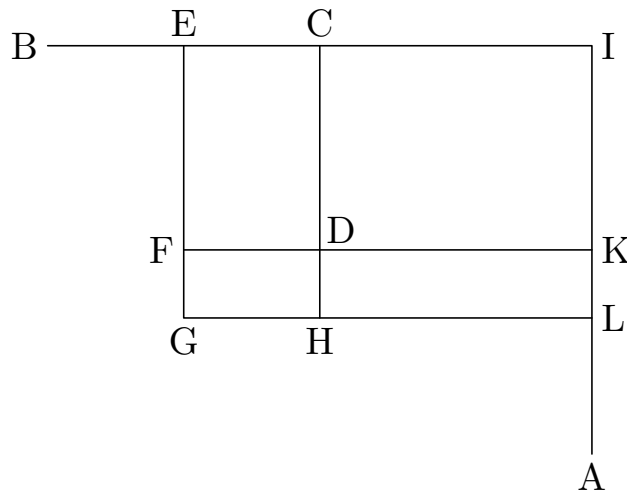
with the Square Root of the Space described; but will still increase from the Beginning to the End of the Motion, and not be the same in any two different Points of the Space described, however near to each other. Let the Velocity increase with the n Power of the Time, that is, let V be as T^n ; and then V will be as $\overline{1+n} \times S^{\frac{n}{1+n}}$ or as $S^{\frac{n}{1+n}}$, putting S for the Space describ'd: Whence it appears that the Velocity will not be the same in any two different Points of the Space described: For it must vary upon the least Change of the Space S , and consequently be different in every different point of AD ; which shews that this Author has been greatly mistaken in imagining that there can be no Motion, no Velocity, in a Point of Space.

IV. The next Questions in the CATECHISM, depending upon each other, run thus. "Let him be asked, whether nothing be not the Product of nothing multiply'd by something? And if so, when the Difference between the Gnomon and Sum of the two Rectangles vanisheth, whether the Rectangles themselves do not also vanish? that is when ab is nothing, whether $A b + B a$ be not also nothing? that is, whether the Momentum of AB be not nothing? And let him be asked what his Momentums are good for when thus they are reduced to nothing?" As to the first of these Questions I agree with him that nothing is the Product of nothing multiply'd by something; but must know what he means by the vanishing of the Gnomon and Sum of the two Rectangles in the second, before I give him a direct Answer. If by vanishing he means that they vanish and become nothings as Areas, I grant they do; but absolutely deny, upon such an Evanscence of the Gnomon and Sum of the two Rectangles by the moving back of the Sides of the Gnomon till they come to coincide with those of the Rectangle, that nothing remains. For there still remain the moving Sides, which are now become the Sides of the Rectangle; into which Sides the Gnomon and the Sum of the two Rectangles are now turned by this retroverted Motion. And as the Gnomon and Sum of the two Rectangles, upon the Evanscence of their Areas by this retroverted Motion, are both converted into the two Sides of the Rectangle AB , so in the Instant of that Conversion, their Motions are exactly the same; or the Motion of the Gnomon is the same with the Sum of the Motions of the two Rectangles, when they evanesce, and are converted into the two Sides of the Rectangle AB .

If a Point moves forward to generate a Line, and afterwards the same Point moves back again to destroy the Line with the very same Degrees of Velocity, in all Parts of the Line which it had in those Parts when moving forward to generate it; in the Instant the Line vanishes as a Length, nothing of a Line will remain; but still the generating Point will remain, together with the Velocity it had at the very Beginning of its Motion. And the Case is the very same with respect to a Rectangle increasing by the Motion of its Sides: For upon the Evanscence of a generated Gnomon, there still remain the Sides of the Rectangle into which the Gnomon by its Evanscence is converted, together with the Velocities of those Sides; that is, when the Gnomon evanesces there still remains $A b + B a$.

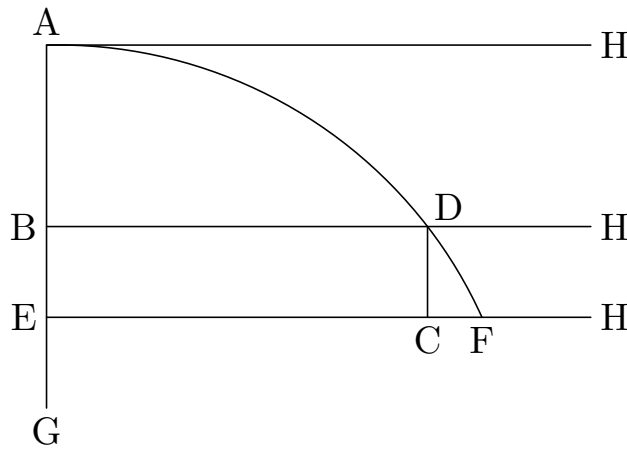


To make this more evident to Sense; let the Point which generated the Line AD, move back again from D to C with the very same Degrees of Velocity in every Part of the Increment CD which it had in that Part in moving forward from C to D; and when the Increment CD evanescs and becomes nothing as a Line by being converted into the moving Point in C; there will remain that Point together with its Motion in C; for neither the Point nor its Motion in C is destroy'd by the Evanescence of the Line CD.



If the Rectangle CDK be generated by the Motions of two indefinite right Lines IB and IA, of IB down IA and of IA along IB, and from a continuance of the Motions of these Lines, be increased by the Gnomon CGK; and if the generating Lines return from L and E to K and C, with the very same Degrees of Velocity in the several Points of LK and EC which they had in those Points in moving from K and C to L and E; it is evident, when the Gnomon CGK or the Increment of the Rectangle CDK evanescs and becomes nothing as an Area, by being converted into the Sides DK and DC; that there will remain those Sides and their Velocities in D; neither the Sides and their Velocities of those Sides being destroy'd by the Evanescence of the Area of the Gnomon. But perhaps he will say, that when the Gnomon evanescs it turns into something more than the Sum of the Sides DK and DC: For the Rectangles LD and FC turn into those Sides, and the Rectangle FH turns into a Point at D; and therefore the Motion of the Gnomon in the Instant it evanescs, will become the Sum of the Motions of the Sides DK and DC, and of the Point in D; that is, putting A and B for DK and DC, and a and b for the Velocities in D towards F and H, and p for a Point; it will be equal to $Ab + Ba + ap + bp$: Let it be so, and yet $\overline{a + b} \times p$ will be nothing when compared with $Ab + Ba$, nor make the least Augmentation to that Sum when added to it.

For let a be to b as 3 to 2, and $Ab + Ba + ap + bp$ will be $2A + 3B + 5p$: But $5p$ or five Points added to a Line, make nothing but the Line itself; a Point being no Part of a Line; and consequently not in the least increasing its Length when added to it: And therefore the Motion of the Gnomon when it evanesces and turns into the Sides of the Rectangle CDK, will be $Ab + Ba$, as Sir *Isaac Newton* makes it.



If the Curvilineal Area A B D be generated by the Motion of any indefinite Line A H down A G, while a point setting out from A moves in the Line A H from A towards H; and if from a Continuance of these two Motions that Area be increased by the Augment B D F E, consisting of the Rectangle B C and of the Triangle C F D; the Motion with which the Increment B D F E begins to be generated, is the Sum of the two Motions towards C and H in the Point D; which Sum, putting b for the Velocity towards C, a for the Velocity towards H and p for a Point, will be $\overline{BD} \times b + ap$. Now suppose the Ordinate EF to move back to BD, and the Point F to move back in the Line HE to C, and that in moving back they have the same Degrees of Velocity in every Point of EB and EH, which they had before in moving forward; it is manifest, that when the Augment B D F E vanishes and becomes nothing as an Area, by being converted into the Ordinate BD, and into a Point at the Extremity of that Ordinate; there will remain BD and the Point in D, together with their Velocities in that Point; for the Motions of that Line and Point, are not destroy'd by the Evanescence of the Area B D F E. In the Instant EF coincides with BD, and the point F with D, the Area B D F E will be converted into $BD + p$, and its Motion will become $BD \times b + pa$, as before. But pa , being only so many points as a contains Units, is really and truly nothing with respect to $BD \times b$; and consequently does not at all increase it when added to it. And therefore the *Fluxion* of the curvilineal Area A B D will be $BD \times b$; or as BD if b be given, or the line A H move down A G with a Velocity which is exactly the same in every Part of G A.

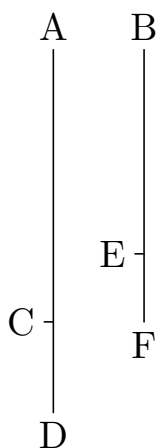
Hence it appears, that if mathematical Quantities be increased in equal Times by Motion, their isochronal Increments must be made to vanish by a Retroversion of the Motion, before we can obtain the Motions with which they vanish, or begin to be generated; that is, before we can obtain the *Fluxions* of the Quantities, the Name given by Sir *Isaac Newton* to those Motions. So then, this Author has been much out in supposing that upon the Evanescence of the Gnomon C G K, or of the curvilineal Figure B D F E, the Momentum or *Fluxion* of the Rectangle CDK, or of the area A B D, vanishes. Consequently, he has been greatly mistaken

in every one of these Questions.

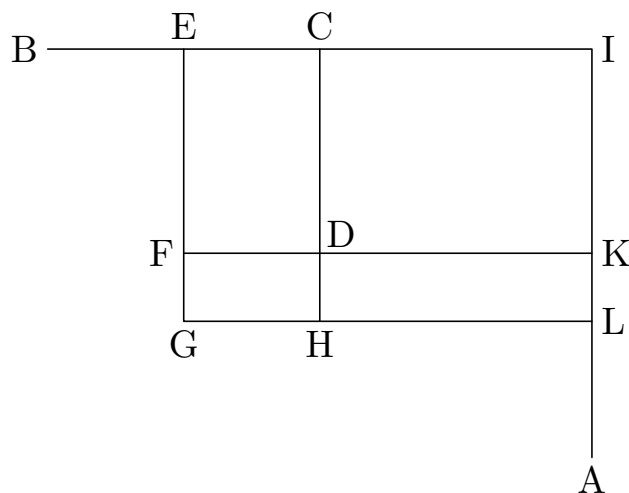
V. But he goes on. “I wish he were asked to explain the Difference between a Magnitude infinitely small and a Magnitude infinitely diminished. If he saith there is no Difference: Then let him be further asked, how he DARES to explain the Method of Fluxions by the *Ratio* of Magnitudes infinitely diminish’d, when Sir *Isaac Newton* hath expresly excluded all Consideration of Quantities infinitely small? If this able Vindicator shou’d say that Quantities infinitely diminish’d are nothing at all, and consequently that, according to him, the first and last *Ratios* are Proportions between nothings, let him be desired to make Sense of this, or explain what he means by *Proportion between nothings*. If he shou’d say the ultimate Proportions are the *Ratios* of mere Limits, then let him be asked how the Limits of Lines can be proportioned or divided?

As all this Part of the CATECHISM relates to the Measures of Fluxions by the first and last *Ratios* of isochronal increments generated and destroy’d by Motion, so I have taken it together, and shall answer the whole in one Section.

Neither Sir *Isaac Newton* nor I have said, that *Fluxions* are measured by the Proportions of Magnitudes infinitely small, nor by the Proportions of any Magnitudes whatever generated in equal Times; but that *they are measured by the first or last Proportions of isochronal Increments generated or destroy’d by Motion*; which Proportions are the *Ratios* with which such Increments begin to exist before they have acquired any Magnitude, or with which they cease to exist and vanish after they have lost all Magnitude. These *Ratios* subsist when the isochronal Increments have no Magnitude, for as much as the Motions subsist with which those Increments, *just now, in this very Instant*, begin or cease to exist; to which Motions these *Ratios* are proportional.



For Example, the first or last *Ratio* of the Increments CD and EF generated in equal Times, has a real Existence; for as much as it is equal to the *Ratio* of the Motions in C and E, which are the Motions subsisting in those Points when the isochronal Increments *just* begin or cease to exist. The Existence of the Motions preserves the Existence of *these Ratios* of the Increments CD and EF. If the *Ratio* of the Motions in C and E, be that of 4 to 3; the first or last *Ratio* of the isochronal Increments CD and EF, will likewise be that of 4 to 3, even tho’ no Part of those Increments has any Existence.



The Motions in C and E, are as the moving Quantities and Velocities taken together; or as two Points and their Velocities taken together; or as the Velocities, all Points being equal. And the first or last *Ratio* of the isochronal Increments C D and E F, is compounded of the first or last *Ratio* of these Spaces, and of the *Ratio* of the moving Quantities. For the Velocities in C and E being in the first or last *Ratios* of these isochronal Spaces, the Motions, which are as the moving Quantities and Velocities taken together, will be as the same moving Quantities and the first or last *Ratio* of the isochronal Spaces taken together. If Q and q denote the moving Quantities in C and E, V and v their Velocities, S and s the isochronal Spaces C D and E F, and $\dot{S}$ and $\dot{s}$ the first or last *Ratio* of those isochronal Spaces, then Q V will be to q v, as Q $\dot{S}$ to q $\dot{s}$; and in this case V will be to v, as $\dot{S}$ to $\dot{s}$, because Q and q are equal.

Again, the first or last *Ratio* of the isochronal Spaces F D and D H in the augmented Rectangle E G L (See the *Figure*) has a real Existence; for as much as it is equal to the *Ratio* of the two Motions of two points in D, of one towards F, and the other towards H; which Motions, subsisting when the isochronal Spaces F D and D H are nothing, preserve the Existence of the first or last *Ratio* of these Spaces, or keep it from being a *Ratio of nothings*. If V and v, denote the Velocities in D towards F and H, Q and q the Sides of the Rectangle D K and D C, and $\dot{S}$ and $\dot{s}$ the first or last *Ratio* of the isochronal Spaces F D and D H; then Q v will be to q V, as Q $\dot{s}$ to q $\dot{S}$; but Q v + q V is the Fluxion or Motion of the Rectangle C D K, as I have shewn before; and therefore the Moment or Measure of the Fluxion of the Rectangle will be Q $\dot{s}$ + q $\dot{S}$. This is a full and clear Answer to this Part of the CATECHISM, and shews that its Author has been greatly mistaken in supposing that I explained the Doctrine of Fluxions by the *Ratio of Magnitudes infinitely diminish'd*, or by *Proportions between nothings*.

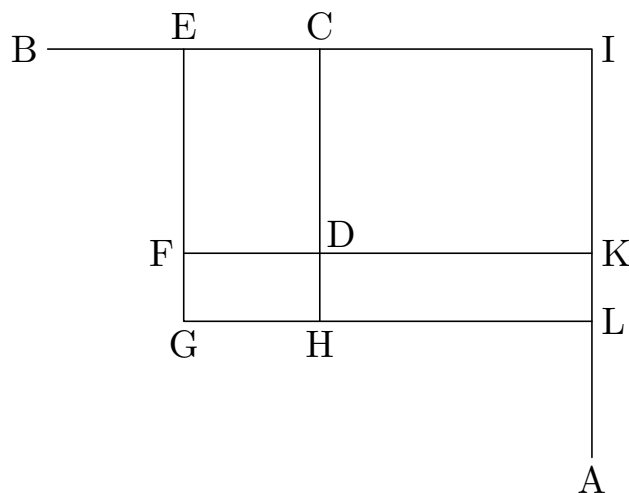
VI. I come now to the last Part of the CATECHISM, which stands thus. "I shall propose one single Expedient by which his Disciples (whom it most concerns) may soon satisfy themselves, whether this Vindicator really understands what he takes upon him to vindicate. It is in short, that they wou'd ask him to explain the second, third, or fourth Fluxions upon his Principles. Be this the Touchstone of his Vindication: If he can do it, I shall own my self much mistaken: If he cannot it will be evident that he was much mistaken in himself, when he *presumed* to defend Fluxions without so much as knowing what they are. So having put

the Merits of the Cause on this Issue, I leave him to be tried by his Scholars.”

I do not wonder that this Author shou’d have no clear Ideas or Conceptions of second, third or fourth Fluxions, when he has no clear Conceptions of the common Principles of Motion, nor of the first and last *Ratios* of the isochronal Increments of Quantities generated and destroy’d by Motion. For Fluxions, according to Sir *Isaac Newton*, are the *Motions* with which the isochronal Increments of Quantities begin or cease to exist, or the Motions of the generating Quantities in the very Limits or Extremities of the *Fluents*: Thus the Fluxions of Solids are the Motions of Surfaces; the Fluxions of Surfaces, the Motions of Lines; the Fluxions of Lines, the Motions of Points; and the Fluxions of Points are nothing, for Points in their own Nature are invariable, and therefore incapable of being generated or increased by Motion: And if the first Fluxions of Quantities be Motions, it follows, that the Mutations of these Motions and the Mutations of those Mutations, which are the second and third Fluxions of the Quantities, must likewise be Motions.

First, second and third Fluxions do really exist, and may be clearly and distinctly conceiv’d by attending to the Motions of the several Parts of a Cube, namely, of its Surfaces of their Lines and Points, in the Instant it begins to be increas’d by Motion. For if A denotes the Side of a Cube generated by an uniform Motion, whose Velocity is express’d by a ; the first Fluxion of the Cube, according to these Principles, will be expressed by $3aA^2$; its second Fluxion, which is the Fluxion of $3aA^2$, will be expressed by $6a^2A$, or by $6aA \times a$; its third Fluxion, which is the First Fluxion of $6a^2A$, will be express’d by $6a^3$ or by $6a^2 \times a$; and its fourth Fluxion will be nothing. But all these Fluxions or Motions do exist, and may be clearly and distinctly conceived in the Motion of a Cube, at the very End of its Generation, or at the very Beginning of its Augmentation, by Motion; for it begins to be augmented by the Sum of the Motions of three of its Squares comprehending any one of its solid Angles, each of which Squares being denoted by A^2 and their Velocity outward by a , the Fluxion of the Cube or the Motion with which it begins to increase or to be enlarged, will be $3aA^2$; and this is the first Fluxion of the Cube; and the three moving Squares begin to be augmented, in the very same Instant wherein the Cube begins to enlarge, each by the Sum of the Motions of its two adjoining Sides, and consequently the Motions of those Sides to augment the cubic Surface, will be express’d by $6aA$, but that Surface at the same Instant of Time moves outward to augment the Solid with a Velocity which is also denoted by a , and therefore the whole Motion of the six moving Sides of the three Squares for increasing or enlarging the Cube, will be expressed by $6aA \times a$ or by $6a^2A$; and this is the second Fluxion of the Cube; and when the three moving Squares begin thus to increase, sideways and outwards, for the Enlargement of the Cube, their six moving Sides begin to be augmented by the Motion of six Points; and the common Velocity of those Points in order to increase the Sides of the Squares, being the same with the Velocity of those Sides to increase the Cubic Surface, and with the Velocity of that Surface to augment the Solid; the whole Motion with which those Points begin to enlarge the Cube, will be express’d by $6a^3$ or by $6a^3p$; and this is the third Fluxion of the Cube: These three kinds of Motion do all necessarily exist and may be clearly and distinctly conceived in the System of Motion whereby a Cube begins to be augmented: And therefore the first, second and third Fluxions of Quantities may be distinctly conceived, and fully explained upon the Principles of Sir *Isaac Newton*.

As first, second and third Fluxions are explained by the several Motions necessarily existing in the very Instant a Cube begins to be augmented, so they may likewise be explain’d



and distinctly comprehended, by considering the nascent or evanescent Increments of the several Parts of a Cube, generated by Motion; provided always that by nascent or evanescent Increments be understood not generated Increments of any Magnitude whatever, but only such Quantities or Magnitudes as are proportional to and Consequently Measures of the Motions with which those isochronal Increments begin or cease to exist. For if a , which before denoted Velocity, be now put for the first or last *Ratio* of the Space described by that Velocity in a given Time; $3aA^2$ will denote the nascent Increment of the Cube generated by the Motion of three of its Squares comprehending any one of its solid Angles; and $6aA$ will express the Sum of the nascent Increments of the three moving Squares, which Sum multiply'd into a will be the Increment of the nascent Solid $3aA^2$; consequently $6a^2A$ will be the second nascent Increment of the Cube; and the nascent Increment of $6Aa$, or of six Rectangles each denoted by Aa , will be $6a^2$, which multiply'd into a gives $6a^3$ for the nascent Increment of $6a^2A$, and therefore $6a^3$ is the third nascent Increment of the Cube: All this may be clearly conceiv'd and made evident to Sense by the *Figure*. For let CDK represent one of the three moving Squares comprehending any of the solid Angles of a Cube increasing by Motion, and then three times CDK multiply'd into a , or $3aA^2$, will expound the first nascent Increment of the Cube; and $3FC + 3LD$ or $6Aa$ (which is the nascent Increment of the three moving Squares) multiply'd into a , will be the second nascent Increment of the Cube; and the Rectangle FH or a^2 being the nascent Increment of the Rectangle FC or LD, and $2FH$ or $2a^2$ the nascent Increment of $FC + LD$ or of $2Aa$, and $6a^2$ the nascent Increment of $6Aa$, it follows that $6a^2$ multiply'd into a , or that $6a^3$, will be the nascent increment of $6Aa^2$, and consequently the third nascent Increment of the Cube. And these three distinct Orders of Increments, all begin to exist together, in the very Instant the Cube begins to be augmented by Motion.

This may serve as an Answer to the last Part of the CATECHISM, concerning the Author's *Touchstone* of my *Vindication*: Whether he will *own himself mistaken* I know not; but I think he ought after his unjust and shameful Treatment of Sir *Isaac Newton*; who in *the Introduction to his Quadrature of Curves*, in *the second Lemma of the second Book*, and in *the Scholium to the first Section of the first Book of his Principles of Philosophy*, has deliver'd his Doctrine of *Fluxions* in so clear and distinct a Manner, without the least *Inconsistency* in

Terms or Arguments, that one would have thought it impossible for any Person not to have understood him, particularly for his Author, who says, *he had long and maturely considered those Principles, and taken as much pains as any Man living to understand and make Sense of them.*

I have now done with the CATECHISM; but beg leave before I conclude this Paper, in order to prevent my being CATECHISED any more by this Author, to give the World a short Account of some Part of my Faith in Religion. I believe that there is one supreme, incorporeal, ever-living, intelligent and omnipresent Being, called GOD, who made and governs the World. I believe that GOD is endued with infinite Power, Knowledge, Wisdom and Goodness; and that to *deny* or *limit* any one of these Attributes, is in Effect to *deny* a GOD. I believe, that to say GOD *cannot* create Spirits with a Power, inherent in themselves and resulting from their own Frame and Make, of perceiving and knowing Things of a quite different Nature from their own, by Ideas and Sensations; is in Effect to *deny* a GOD; for as much as by this Principle his Almighty Power is denied. And lastly, I believe that this Supreme Being has revealed his Will to Mankind by *Moses*, the Prophets, *Jesus Christ* and the Apostles; and that the Doctrine by them deliver'd is therefore divine, and cannot be altered by any Power or Authority upon Earth, nor even by an Angel from Heaven, who is pronounced accursed shou'd he preach any other *Gospel* than what is delivered.

FINIS.